Introduction to causal graphical models

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(conditional) Independence

Conditional independence of random variables For a distribution P, X and Y are independent conditioned on Z, noted $X \perp\!\!\!\perp_P Y \mid Z$, *iff*:

$$P(X, Y \mid Z) = P(X \mid Z) P(Y \mid Z)$$

$$\text{or } P(X \mid Y, Z) = P(X \mid Z) \text{ if } P(Y, Z) > 0$$

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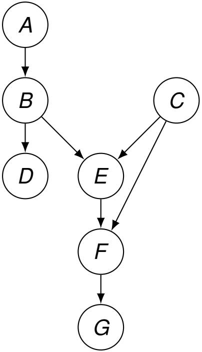
Properties

Symmetry: $X \perp\!\!\!\perp_P Y \mid Z \implies Y \perp\!\!\!\perp_P X \mid Z$
Decomposition: $X \perp\!\!\!\perp_P Y, W \mid Z \implies X \perp\!\!\!\perp_P Y \mid Z$
Weak union: $X \perp\!\!\!\perp_P Y, W \mid Z \implies X \perp\!\!\!\perp_P Y \mid Z, W$
Contraction: $X \perp\!\!\!\perp_P Y \mid Z \;\&\; X \perp\!\!\!\perp_P W \mid Z, Y \implies X \perp\!\!\!\perp_P Y, W \mid Z$
Intersection: $X \perp\!\!\!\perp_P W \mid Z, Y \;\&\; X \perp\!\!\!\perp_P Y \mid Z, W \implies X \perp\!\!\!\perp_P Y, W \mid Z$

Basic graph concepts

Consider the following graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$:



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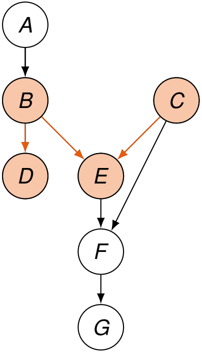

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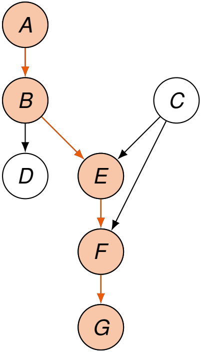

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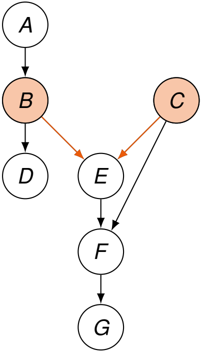
Parents: $Pa(E) = \{B, C\}$



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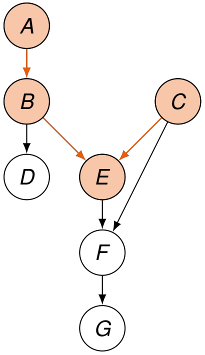

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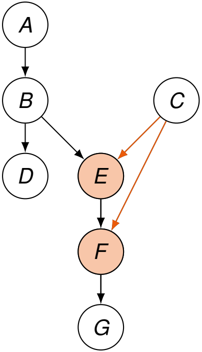

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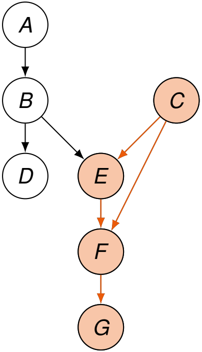

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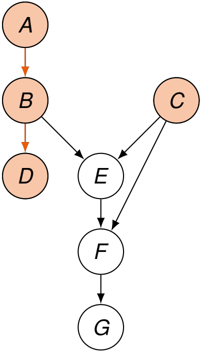
Non-descendants: $Nd(E) = \{A, B, C, D\}$



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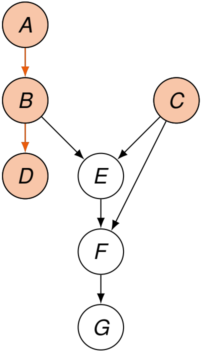
Ancestral sets: a subset of nodes $\mathcal{S}$ is ancestral (or upward-closed) if $\forall S \in \mathcal{S}, An(S) \subseteq \mathcal{S}$



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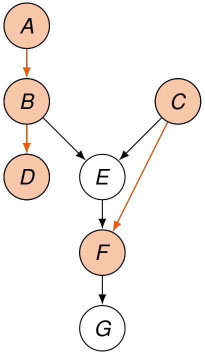
Induced subgraph $\mathcal{G}[\mathcal{S}]$: $\mathcal{G}[\{B, C, D, F\}]$



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Bayesian networks and compatibility

Compatibility We say that a distribution $P(\mathcal{V})$ is compatible with (or Markov relative to) a DAG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ if $P(\mathcal{V}) = \prod_{X \in \mathcal{V}} P(X \mid Pa(X))$.

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Bayesian network A DAG (directed acyclic graph) $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a Bayesian network *iff* there exists a joint distribution $P(\mathcal{V})$ that is compatible with $\mathcal{G}$.

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Decomposing with respect to ancestral sets If P is compatible with $\mathcal{G}$ and $\mathcal{S} \subseteq \mathcal{V}$ is an ancestral set, then
$P(\mathcal{S})$ is compatible with $\mathcal{G}[\mathcal{S}]$ (*i.e.*, $P(\mathcal{S}) = \prod_{S \in \mathcal{S}} P(S \mid Pa(S))$)
and $P(\mathcal{V} \backslash \mathcal{S} \mid \mathcal{S})$ is compatible with $\mathcal{G}[\mathcal{V} \backslash \mathcal{S}]$
(proof on board)

Testing compatibility

Proposition (Ordered Markov condition) P is compatible with $\mathcal{G}$ *iff* in any topological ordering $X_1, \cdots, X_n$ of $\mathcal{V}$, we have that

$$X_i \perp\!\!\!\perp X_1, \cdots, X_{i-1} \mid Pa(X_i) \qquad \text{for } i = 1, \cdots, n$$

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Proposition (Parental Markov condition (also known as local Markov condition)) P is compatible with $\mathcal{G}$ *iff* $\forall X \in \mathcal{V}$ we have that

$$X \perp\!\!\!\perp Nd(X) \mid Pa(X)$$

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Proposition (Conditioning on common ancestors) For disjoint $\mathcal{X}, \mathcal{Y}, \mathcal{Z} \subseteq \mathcal{V}$, if $An(\mathcal{X}) \cap An(\mathcal{Y}) \subseteq \mathcal{Z}$ and $An(\mathcal{Z}) \subseteq (\mathcal{Z})$, then

$$P(\mathcal{X}, \mathcal{Y} \mid \mathcal{Z}) = P(\mathcal{X} \mid \mathcal{Z}) P(\mathcal{Y} \mid \mathcal{Z}) \text{ (i.e., } \mathcal{X} \perp\!\!\!\perp_P \mathcal{Y} \mid \mathcal{Z})$$

in any distribution P compatible with $\mathcal{G}$
(proof on board)

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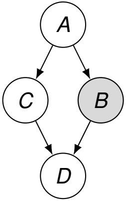
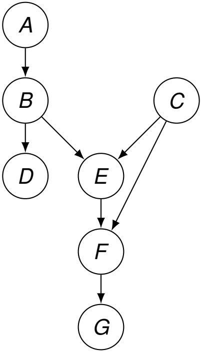
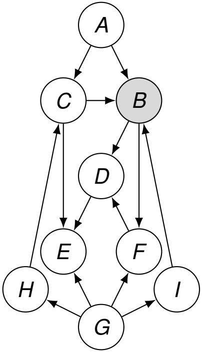
Reading conditional independencies in graphs

$A \overset{?}{\perp\!\!\!\perp}_{\mathcal{P}} D \mid B$



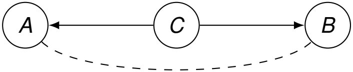


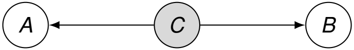


Basic structures

Basic structures

Fork





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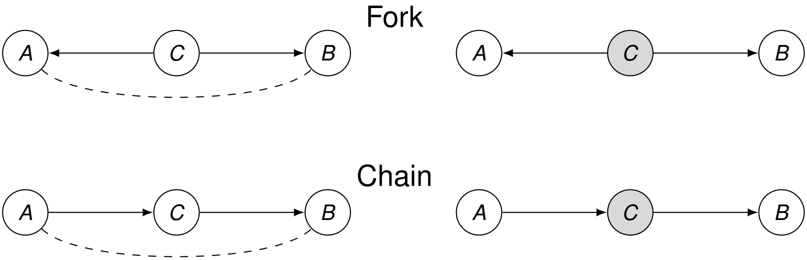

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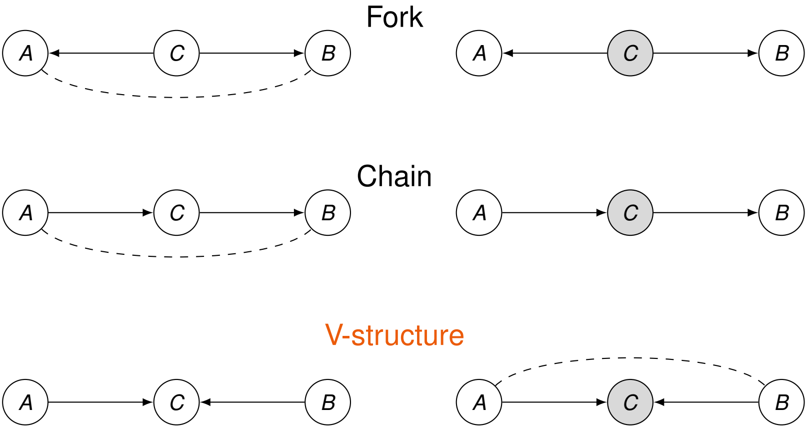
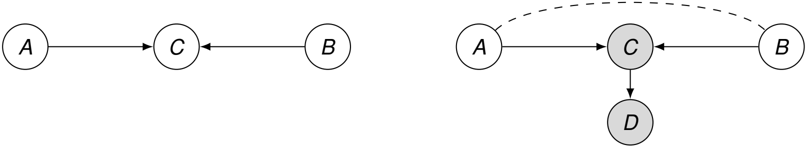


Basic structures

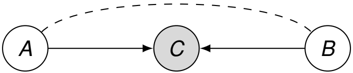
Fork

Chain

V-structure



Artificial correlation in V-structures



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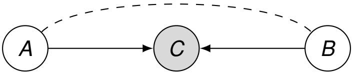

Example 1: $A = \begin{cases} Chicken \\ Other \backslash \{Rooster\} \end{cases} B = \begin{cases} Rooster \\ Other \backslash \{Chicken\} \end{cases} C = A \& B = \begin{cases} Chick \\ Other \end{cases}$

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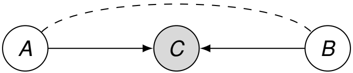

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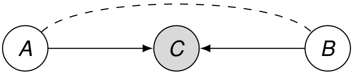

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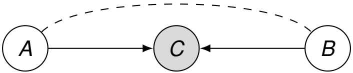

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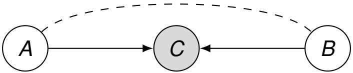
Example 2: $A, B \sim U(-1, 1)$ $\quad \xi_c \sim N(0, \frac{1}{2})$ $\quad C = 2AB + \xi_c$

Artificial correlation in V-structures



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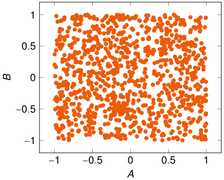
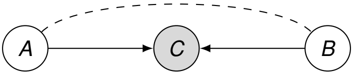


$Corr(A; B) = 0.002$

Artificial correlation in V-structures



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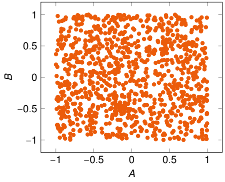

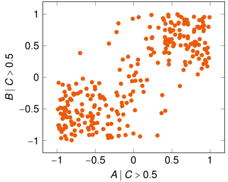

$Corr(A; B) = 0.002$ $Corr(A; B \mid C > 0.5) = 0.8$

Blocked paths

Collider[1] A triple such that $X \rightarrow Z \leftarrow Y$. If the two parent vertices are not adjacent, the collider is a v-structure (also called unshielded collider or immorality)

[1] We also refer to Z as the collider

Blocked paths

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Active and blocked paths A path is said to be blocked by a set of vertices $\mathcal{Z} \in \mathcal{V}$ if:

- it contains a chain $A \rightarrow B \rightarrow C$ or a fork $A \leftarrow B \rightarrow C$ and $B \in \mathcal{Z}$, or
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A path that is not blocked is active. A path is active if every triple along the path is active, and blocked if a single triple is blocked

[1] We also refer to Z as the collider

d-separation

d-separation (also known as the global Markov condition) Given disjoint sets $\mathcal{X}, \mathcal{Y}, \mathcal{Z} \subseteq \mathcal{V}$, we say that $\mathcal{X}$ and $\mathcal{Y}$ are d-separated by $\mathcal{Z}$ if every path between a node in $\mathcal{X}$ and a node in $\mathcal{Y}$ is blocked by $\mathcal{Z}$ and we write $\mathcal{X} \perp\!\!\!\perp_G \mathcal{Y} | \mathcal{Z}$.

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If one of the above path is not blocked, we say that $\mathcal{X}$ and $\mathcal{Y}$ are d-connected given $\mathcal{Z}$

d-separation characterizes the conditional independencies of distributions compatible with a given DAG

Theorem (probabilistic implications of d-separation)

(i) *Soundness* $\mathcal{X} \perp\!\!\!\perp_G \mathcal{Y} \mid \mathcal{Z} \Rightarrow \mathcal{X} \perp\!\!\!\perp_P \mathcal{Y} \mid \mathcal{Z}$ in every distribution P compatible with $\mathcal{G}$

(ii) *Completeness* If $\mathcal{X} \not\perp\!\!\!\perp_G \mathcal{Y} \mid \mathcal{Z}$, then there exists a distribution P compatible with $\mathcal{G}$ such that $\mathcal{X} \not\perp\!\!\!\perp_P \mathcal{Y} \mid \mathcal{Z}$

Proof in (Pearl, 1988)

d-separation and conditional independence

d-separation characterizes the conditional independencies of distributions compatible with a given DAG

Theorem (probabilistic implications of d-separation)

(i) *Soundness* $\mathcal{X} \perp\!\!\!\perp_G \mathcal{Y} \mid \mathcal{Z} \Rightarrow \mathcal{X} \perp\!\!\!\perp_P \mathcal{Y} \mid \mathcal{Z}$ in every distribution P compatible with $\mathcal{G}$

(ii) *Completeness* If $\mathcal{X} \not\perp\!\!\!\perp_G \mathcal{Y} \mid \mathcal{Z}$, then there exists a distribution P compatible with $\mathcal{G}$ such that $\mathcal{X} \not\perp\!\!\!\perp_P \mathcal{Y} \mid \mathcal{Z}$

Proof in (Pearl, 1988)

d-separation and conditional independence

d-separation characterizes the conditional independencies of distributions compatible with a given DAG

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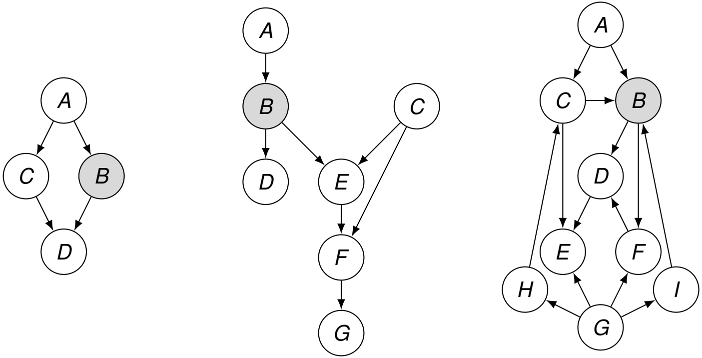
Reading conditional independencies in graphs using d-separation

$A \overset{?}{\perp\!\!\!\perp}_{\mathcal{P}} D \mid B$



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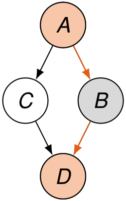

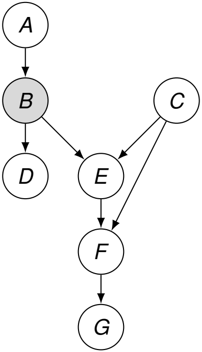

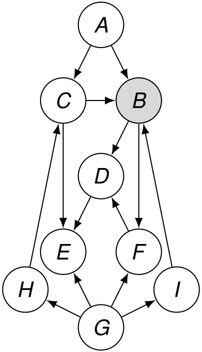

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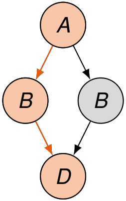

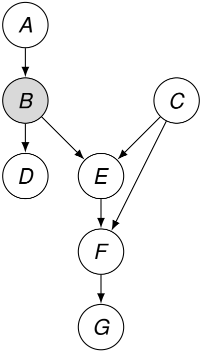

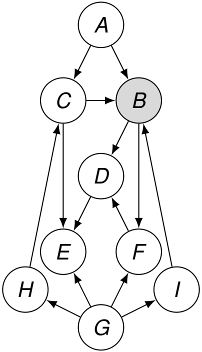

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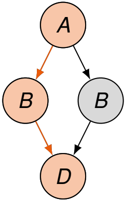

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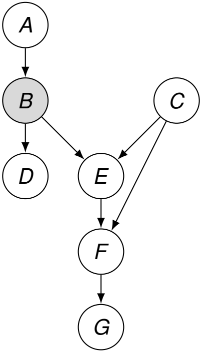

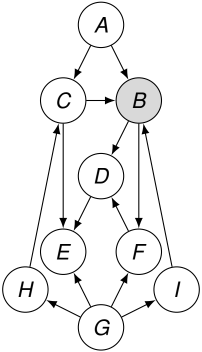

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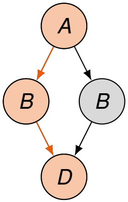

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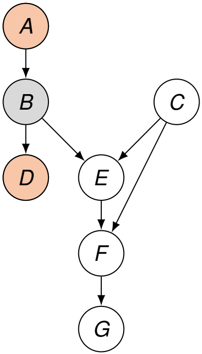

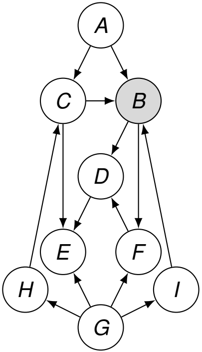

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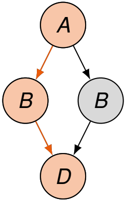

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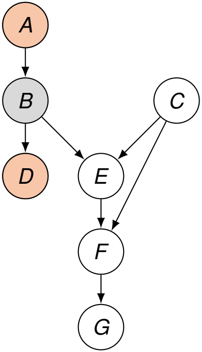

All paths are blocked

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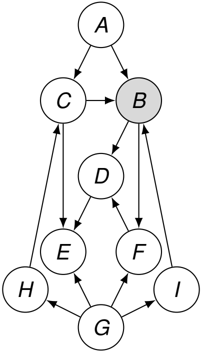

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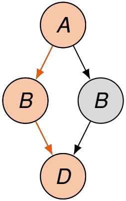

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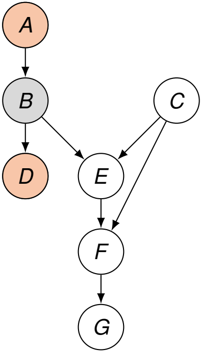

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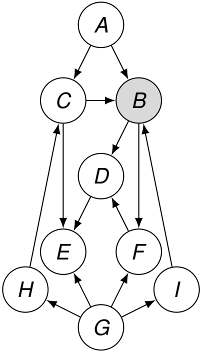

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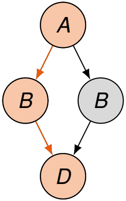

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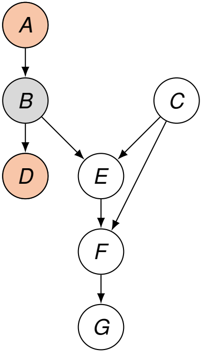

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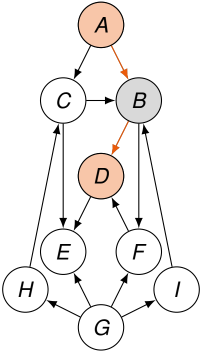

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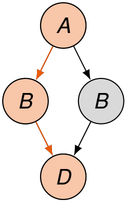

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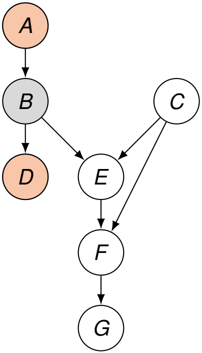

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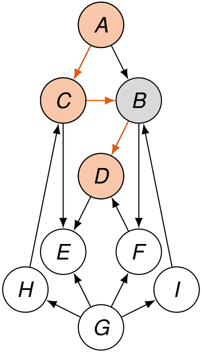

Reading conditional independencies in graphs using d-separation

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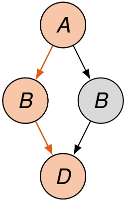

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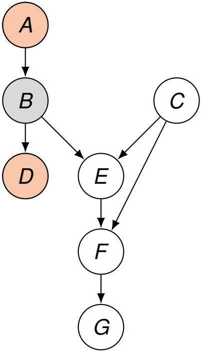

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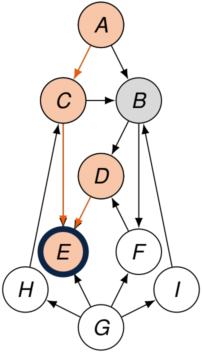

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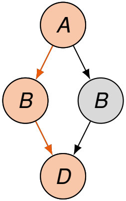

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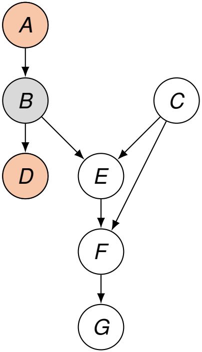

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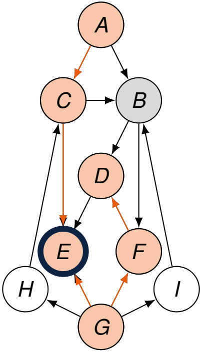

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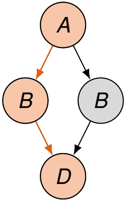

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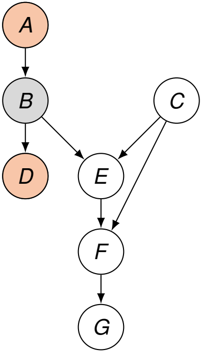

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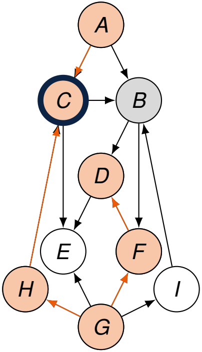

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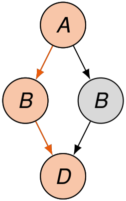

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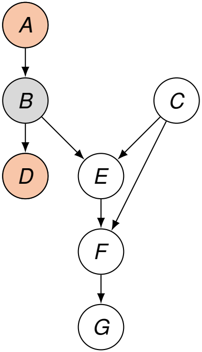

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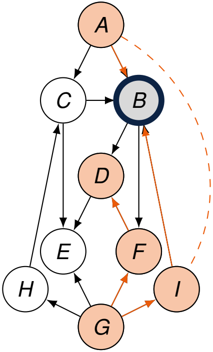

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Reading conditional independencies in graphs using d-separation

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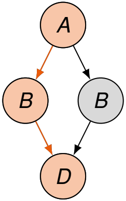

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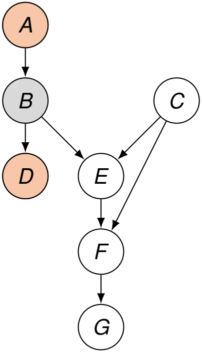

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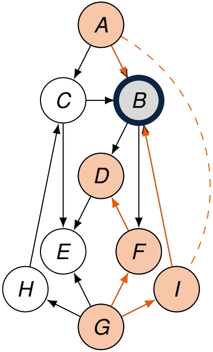
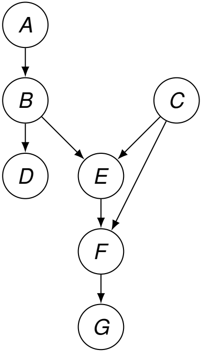


$< A, I, G, F, D >$ is not blocked

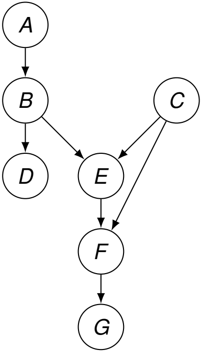
$\Longrightarrow \; A \overset{?}{\perp\!\!\!\perp}_{\mathcal{P}} D \mid B$

More examples



- $B \perp\!\!\!\perp_P G \mid F$?
- $A \perp\!\!\!\perp_P F \mid C, E$?
- $B \perp\!\!\!\perp_P E \mid F$?

More examples



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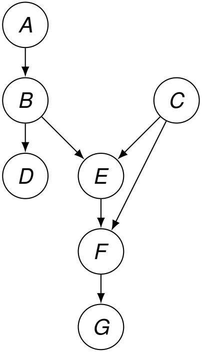

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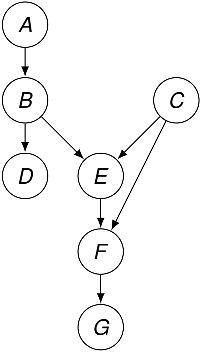

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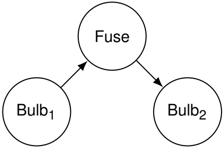
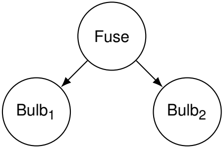
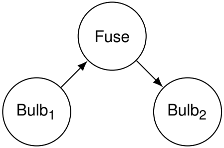
Table of content

Bayesian networks vs causal graph



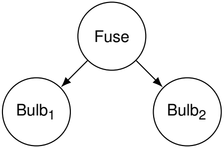


Bayesian networks vs causal graph



$\text{Bulb}_1 \perp\!\!\!\perp_P \text{Bulb}_2 \mid \text{Fuse}$

Bayesian network



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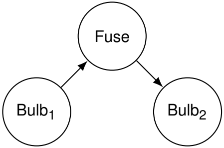
Bayesian network

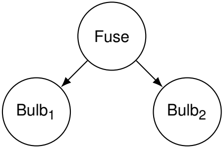
Bayesian networks vs causal graph



$Bulb_1 \perp\!\!\!\perp_P Bulb_2 \mid Fuse$

Bayesian network

Not a causal graph



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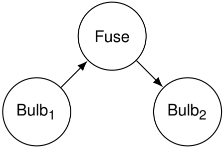
Bayesian network

Causal graph

Bayesian networks vs causal graph



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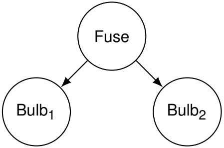
Bayesian network

Not a causal graph



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Bayesian network

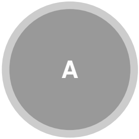
Causal graph

Oracle for conditional independence

Oracle for intervention

Conditioning vs Intervening (1/2)

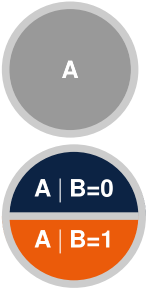
Population



Conditioning vs Intervening (1/2)

Population

Sub-populations



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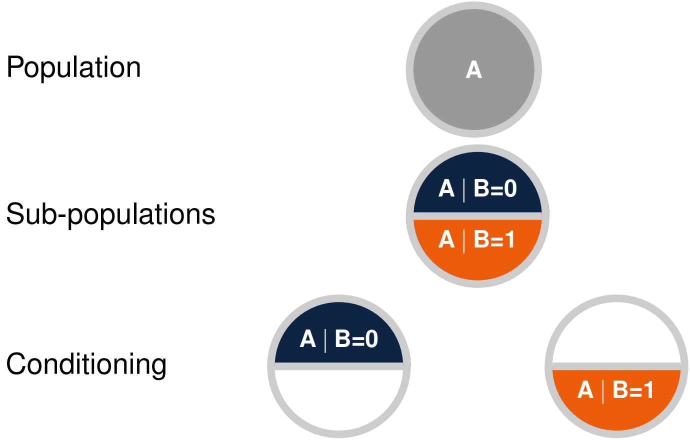

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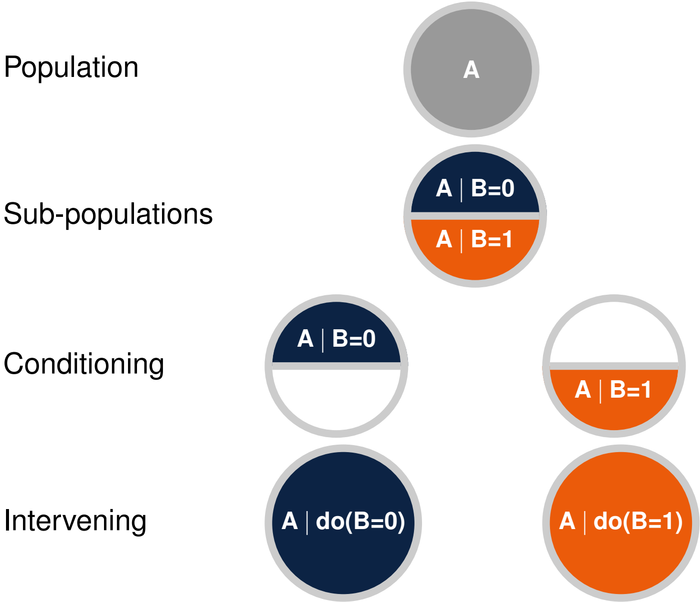
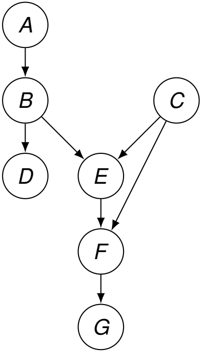
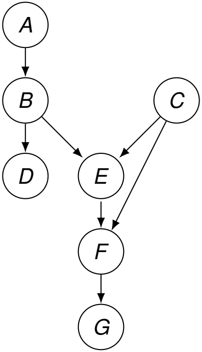


Conditioning vs Intervening (2/2)





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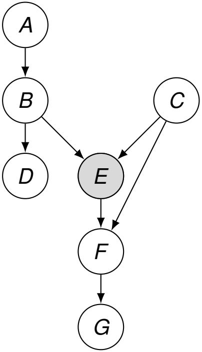

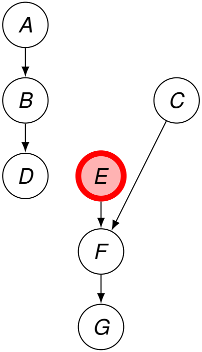

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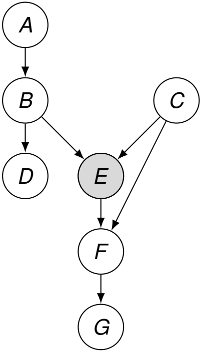

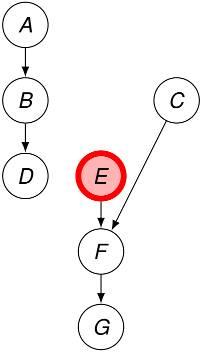

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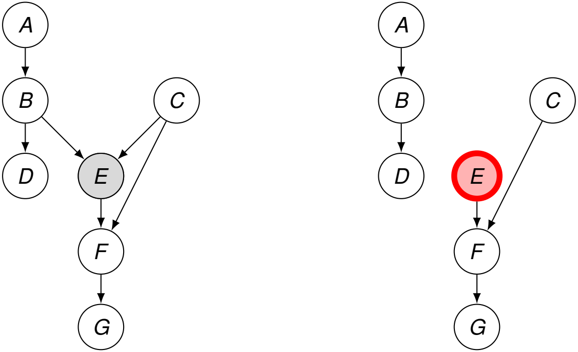

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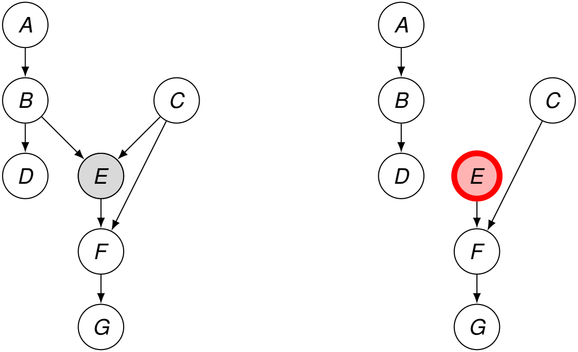

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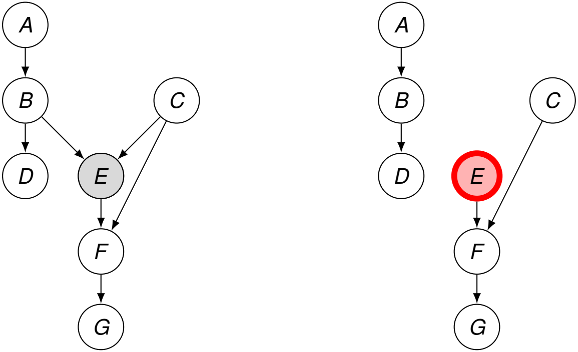

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From association to causation (1/2)

Reminder: parental Markov condition

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Causal Markov condition

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From association to causation (2/2)

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Truncated factorization (also known as the manipulation theorem) If we intervene on a subset $S \subset \mathbf{V}$, then

$$\Pr_{\{S=s\}}(\mathbf{V}_1 = \mathbf{v}_1, \cdots, \mathbf{V}_d = \mathbf{v}_d) = \prod_{i \notin S} \Pr(\mathbf{V}_i \mid Pa(\mathbf{V}_i))$$

if $\mathbf{v}_1, \cdots, \mathbf{v}_d$ are values consistant with the intervention, else,

$$\Pr_{\{S=s\}}(\mathbf{V}_1 = \mathbf{v}_1, \cdots, \mathbf{V}_d = \mathbf{v}_d) = 0$$

Causal Bayesian networks

Causal Bayesian network Let $P(\mathcal{V})$ be a probability distribution and let $P(\mathcal{V} \mid do(s))$ denote the distribution resulting from the intervention that sets a subset $\mathcal{S}$ of variables to constants s. Let $\mathcal{P}_*$ denote the set of all interventional distributions $P(\mathcal{V} \mid do(s))$. A DAG $\mathcal{G}$ is said to be a causal Bayesian network compatible with $\mathcal{P}_*$ *iff* $\mathcal{G}$ and $\mathcal{P}_*$ satisfy the truncated factorization.

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Causal discovery

- It is possible to infer a causal graph from observational data?
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Identifiability: The causal effect of an intervention $do(x)$ on a set of variables Y such that $Y \cap X = \varnothing$ is said to be identifiable from P in $\mathcal{G}$ if $P(Y \mid do(x))$ is uniquely computable from $P(\mathcal{V})$.

Table of content

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Linear structural causal model It consists on a set of structural equations of the form:

$$y := \sum_{x \in Pa(y)} \beta_{xy} x + \xi_y$$

where $Pa(y)$ are direct causes of y, ξ_y represent errors due to ommitted factors and β_{xy} which are known as a structural coefficient represents the strength of the causal relation.

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$P(\mathcal{U})$ and $\mathcal{F}$ induce a joint distribution $P(\mathcal{V})$ over $\mathcal{V}$.

Induced graph

Induced graph *The graph $\mathcal{G}$ induced by a structural causal model M has vertices $\mathcal{V}$ and an edge $X_i \rightarrow X_j$ whenever f_j depends on X_i. In addition, $\mathcal{G}$ contains a bidirected edge, denoted $X_i \dashleftarrow\dashrightarrow X_j$, whenever f_i and f_j depend on a common subset of $\mathcal{U}$.*

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Induced distribution in Markovian models

$P(\mathcal{V})$ does not depend on $\mathcal{U}$ in Markovian causal models

$$P(\mathcal{V} \cup \mathcal{U}) = \prod_{i=1}^{n} P(x_i \mid Pa(x_i), u_i) P(u_i)$$

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Example of a Markovian model

$$M : \begin{cases} A := f_a(\xi_a) \\ B := f_b(A, H, \xi_b) \\ C := f_c(A, B, I, \xi_c) \\ D := f_d(C, F, \xi_d) \\ E := f_e(B, G, \xi_e) \\ F := f_f(C, G, \xi_f) \\ G := f_g(\xi_g) \\ H := f_h(G, \xi_h) \\ I := f_i(G, \xi_i) \end{cases}$$

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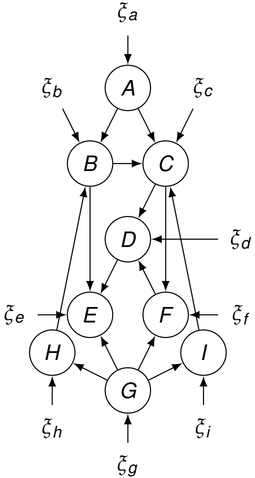

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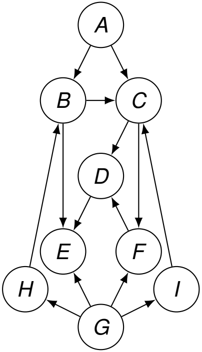

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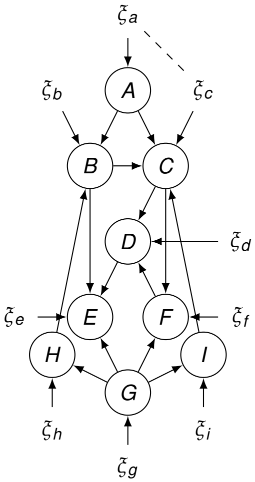

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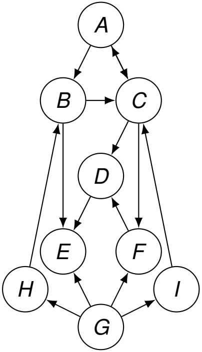

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SCMs and interventions

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Interventional SCM

$$M_c : \begin{cases} A := f_a(\xi_a) \\ B := f_b(A, H, \xi_b) \\ C := c \\ D := f_d(C, F, \xi_d) \\ E := f_e(B, G, \xi_e) \\ F := f_f(C, G, \xi_f) \\ G := f_g(\xi_g) \\ H := f_h(G, \xi_h) \\ I := f_i(G, \xi_i) \end{cases}$$

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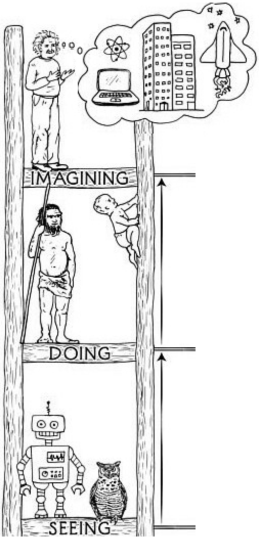
Conclusion



SCMs

Causal graphs

Bayesian networks

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