Tools to Ascend the Ladder of Causation

Causal reasoning

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Causal hierarchy

Some health-related questions

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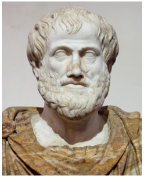
- Was it the aspirin that stopped by headache?

A history of attempts ...

−384 1748 1954 1970

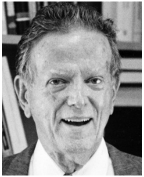
Aristotle



Hume

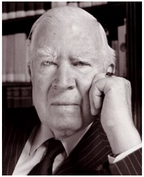
Reichenbach

Suppes

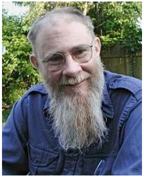


1971 1972 1973 2011

von Wright



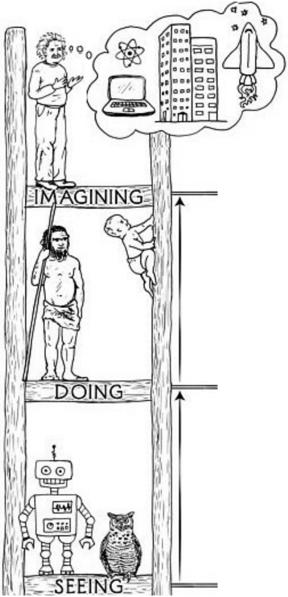
Lewis



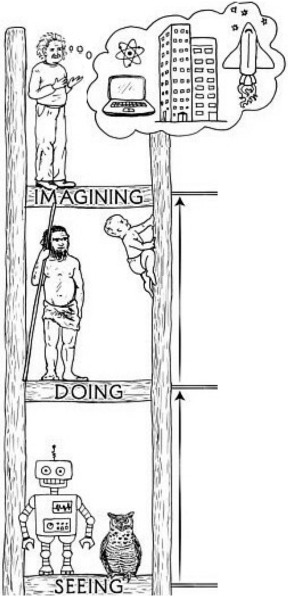
Anscombe

Pearl

Pearl causal hierarchy



Pearl causal hierarchy



- Associations
  - Questions : What if I see ...?
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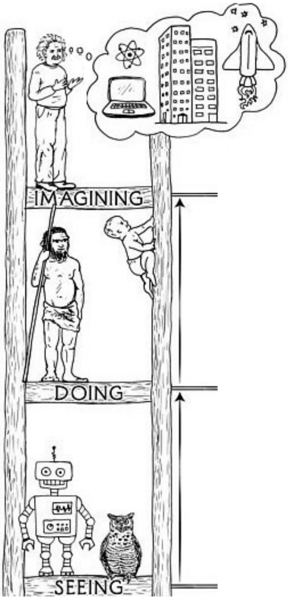
Pearl causal hierarchy



- Interventions
  - Questions: What if I do ...? How?
  - Does exercising reduce cholesterol?
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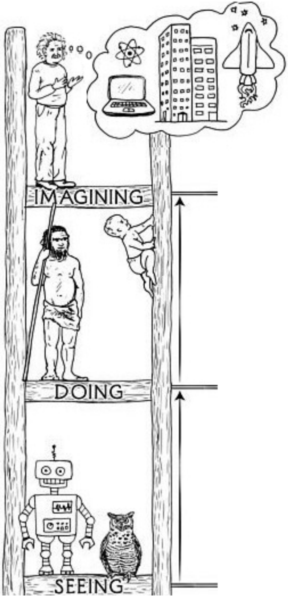

- Counterfactuals
  - Questions: What if I had done ...? Why?
  - Would COVID-19 outcomes in France have changed with a different policy?
  - Was it the aspirin that stopped by headache?
- Interventions
  - Questions: What if I do ...? How?
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Associations

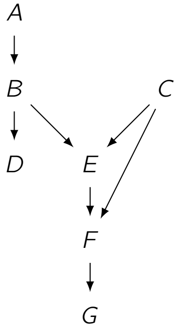
Notations

- Sets: $\mathbb{A} = \{X, Y, Z\}$
- Statistical independence: $\perp\!\!\!\perp_P$
- Statistical dependence: $\not\!\perp\!\!\!\perp_P$

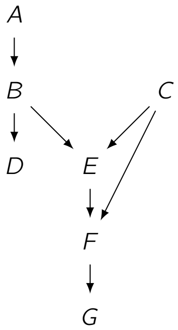
Directed acyclic graphs

A graph $\mathcal{G} = (\mathbb{V}, \mathbb{E})$ is said to be a directed acyclic graph (DAG) if

- $\mathbb{V}$ is the set of nodes (usually each node corresponds to a random variable),
- $\mathbb{E}$ is the set of edges,
- $\forall (X, Y) \in \mathbb{E}$,
  there is an arrow pointing from X to Y,
- there are no directed cycle in $\mathcal{G}$.



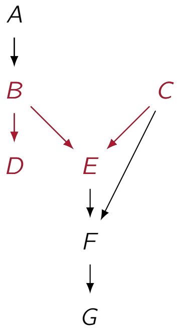
Consider the following DAG $\mathcal{G} = (\mathbb{V}, \mathbb{E})$:



DAGs: basic concepts

Consider the following DAG $\mathcal{G} = (\mathbb{V}, \mathbb{E})$:

Path: $D \leftarrow B \rightarrow E \leftarrow C$



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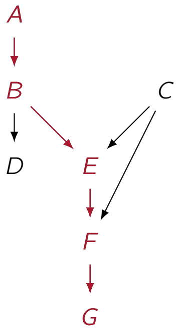

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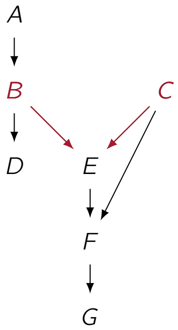

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Parents: $Pa(E) = \{B, C\}$

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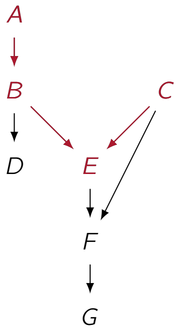

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Ancestors: $An(E) = \{A, B, C, E\}$

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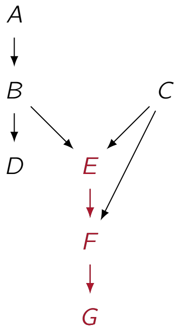

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Descendants: $De(E) = \{E, F, G\}$

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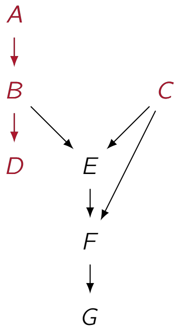

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Non-descendants: $Nd(E) = \{A, B, C, D\}$

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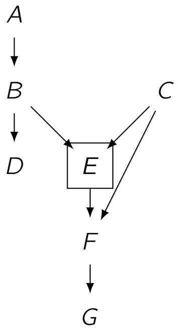

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Conditioning: represented by a square

Python code: DAGs using pyCIPHOD package

```
#Load the pyCIPHOD package
from pyciphod.graphs import DirectedAcyclicGraph
#Define the Directed graph
g = DirectedAcyclicGraph()
g.add_vertices(["A","B","C"])
g.add_directed_edge('B', 'A')
g.add_directed_edge('B', 'C')
g.remove_directed_edge("A", "C")
#Print nodes and edges
print(g.get_vertices())
print(g.get_directed_edges())
#Print parents and children
print(g.get_parents("A"))
print(g.get_children("B"))
```

Bayesian network

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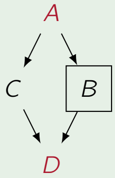
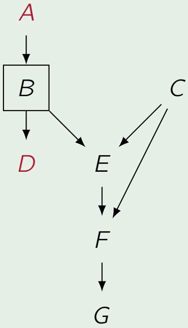
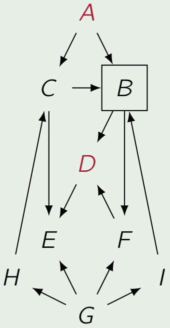
A DAG $\mathcal{G} = (\mathbb{V}, \mathbb{E})$ is a Bayesian network if there exists a joint distribution $P(\mathbb{V})$ that is compatible with $\mathcal{G}$.

Example

$A \overset{?}{\perp\!\!\!\perp}_P D \mid B$







Basic structures

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Fork: contains a common parent

$$A \leftarrow C \rightarrow B \qquad\qquad A \leftarrow \boxed{C} \rightarrow B$$

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$$A \perp\!\!\!\perp_P B$$

$$\begin{tikzcd} A \arrow[r] & C \arrow[d] & B \arrow[l] \\ & \boxed{D} & \end{tikzcd}$$

$$A \overset{?}{\perp\!\!\!\perp}_P B \mid D$$

Artificial correlation in colliders: example 1

Example

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$$A \longrightarrow \boxed{C} \longleftarrow B$$

$$A = \begin{cases} \text{Mother carrier} \\ \text{Mother not carrier} \end{cases} \qquad B = \begin{cases} \text{Father carrier} \\ \text{Father not carrier} \end{cases}$$

$$C = (A \text{ or } B) = \begin{cases} \text{Child carrier} \\ \text{Child not carrier} \end{cases}$$

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If $C = \text{Child carrier} \implies$

$$\begin{cases} \text{If } A = \text{Mother not carrier then } B = \text{Father carrier} \\ \text{If } B = \text{Father not carrier then } A = \text{Mother carrier} \end{cases}$$

Artificial correlation in colliders: example 2

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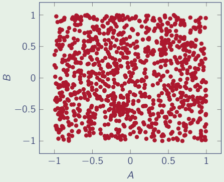

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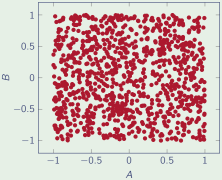

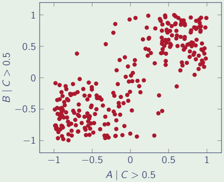

$Corr(A; B) = 0.002$ $\quad Corr(A; B \mid C > 0.5) = 0.8$

Blocked paths

A path is said to be blocked by a set of nodes $\mathbb{Z} \in \mathbb{V}$ if:

- it contains a chain $A \to B \to C$ or a fork $A \leftarrow B \to C$ and $B \in \mathbb{Z}$; or
- it contains a collider $A \to B \leftarrow C$ such that no descendant of B is in $\mathbb{Z}$.

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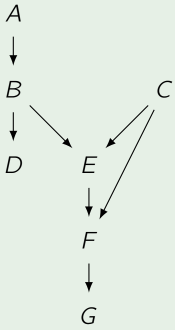
A path that is not blocked is active.

Example



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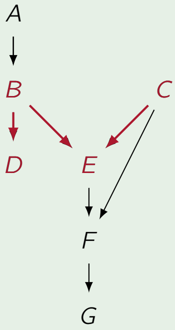
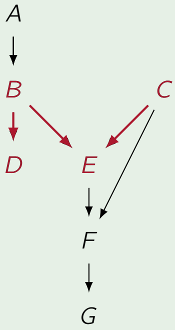


Is the path $< D, B, E, C >$ blocked?

Blocked path: examples

Example



Is the path $< D, B, E, C >$ blocked? Yes

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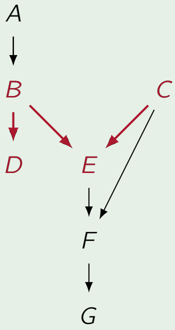

Is the path $< D, B, E, C >$ blocked? Yes
Is the path $< D, B, E, C >$ blocked by E?

Blocked path: examples

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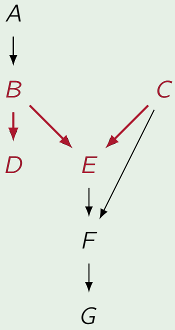

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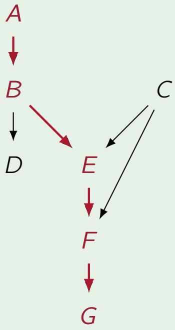

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Blocked path: examples

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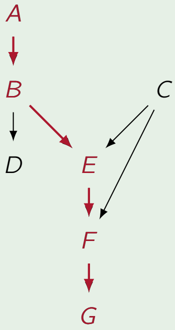

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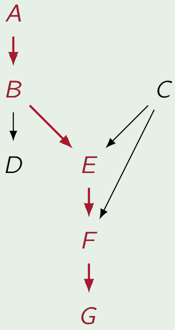

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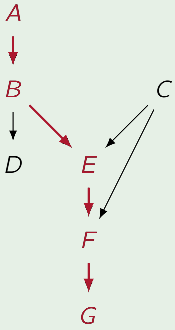

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d-separation

Given disjoint sets $\mathbb{X}, \mathbb{Y}, \mathbb{Z} \subseteq \mathbb{V}$, we say that $\mathbb{X}$ and $\mathbb{Y}$ are d-separated by $\mathbb{Z}$ if every path between a node in $\mathbb{X}$ and a node in $\mathbb{Y}$ is blocked by $\mathbb{Z}$ and we write $\mathbb{X} \perp\!\!\!\perp_{\mathcal{G}} \mathbb{Y} \mid \mathbb{Z}$.

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If one path is not blocked, we say that $\mathbb{X}$ and $\mathbb{Y}$ are d-connected given $\mathbb{Z}$ and we write $\mathbb{X} \not\perp\!\!\!\perp_{\mathcal{G}} \mathbb{Y} \mid \mathbb{Z}$.

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Theorem

$\mathbb{X} \perp\!\!\!\perp_{\mathcal{G}} \mathbb{Y} \mid \mathbb{Z} \Rightarrow \forall P$ compatible with $\mathcal{G}, \mathbb{X} \perp\!\!\!\perp_{P} \mathbb{Y} \mid \mathbb{Z}$

Given disjoint sets $\mathbb{X}, \mathbb{Y}, \mathbb{Z} \subseteq \mathbb{V}$, we say that $\mathbb{X}$ and $\mathbb{Y}$ are d-separated by $\mathbb{Z}$ if every path between a node in $\mathbb{X}$ and a node in $\mathbb{Y}$ is blocked by $\mathbb{Z}$ and we write $\mathbb{X} \perp\!\!\!\perp_{\mathcal{G}} \mathbb{Y} \mid \mathbb{Z}$.

If one path is not blocked, we say that $\mathbb{X}$ and $\mathbb{Y}$ are d-connected given $\mathbb{Z}$ and we write $\mathbb{X} \not\perp\!\!\!\perp_{\mathcal{G}} \mathbb{Y} \mid \mathbb{Z}$.

Theorem

$\mathbb{X} \perp\!\!\!\perp_{\mathcal{G}} \mathbb{Y} \mid \mathbb{Z} \Rightarrow \forall P$ compatible with $\mathcal{G}, \mathbb{X} \perp\!\!\!\perp_{P} \mathbb{Y} \mid \mathbb{Z}$
but $\mathbb{X} \not\perp\!\!\!\perp_{\mathcal{G}} \mathbb{Y} \mid \mathbb{Z} \not\Rightarrow \forall P$ compatible with $\mathcal{G}, \mathbb{X} \not\perp\!\!\!\perp_{P} \mathbb{Y} \mid \mathbb{Z}$

d-separation

Given disjoint sets $\mathbb{X}, \mathbb{Y}, \mathbb{Z} \subseteq \mathbb{V}$, we say that $\mathbb{X}$ and $\mathbb{Y}$ are d-separated by $\mathbb{Z}$ if every path between a node in $\mathbb{X}$ and a node in $\mathbb{Y}$ is blocked by $\mathbb{Z}$ and we write $\mathbb{X} \perp\!\!\!\perp_{\mathcal{G}} \mathbb{Y} \mid \mathbb{Z}$.

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$\mathbb{X} \not\perp\!\!\!\perp_{\mathcal{G}} \mathbb{Y} \mid \mathbb{Z} \Rightarrow \exists P$ compatible with $\mathcal{G}, \mathbb{X} \not\perp\!\!\!\perp_{P} \mathbb{Y} \mid \mathbb{Z}$

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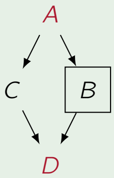
$$\mathbb{X} \perp\!\!\!\perp_{\mathcal{G}} \mathbb{Y} \mid \mathbb{Z} \Rightarrow \mathbb{X} \perp\!\!\!\perp_{P} \mathbb{Y} \mid \mathbb{Z}$$

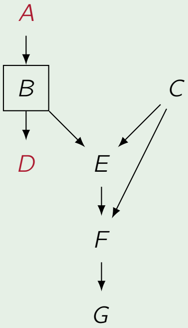
$$\mathbb{X} \not\!\perp\!\!\!\perp_{\mathcal{G}} \mathbb{Y} \mid \mathbb{Z} \Rightarrow \mathbb{X} \overset{?}{\perp\!\!\!\perp}_{P} \mathbb{Y} \mid \mathbb{Z}$$

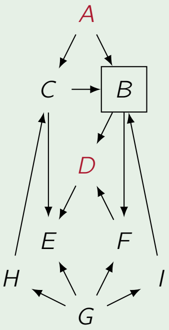
Reading conditional independencies using d-separation

Example

Is $A \perp\!\!\!\perp_P D \mid B$?







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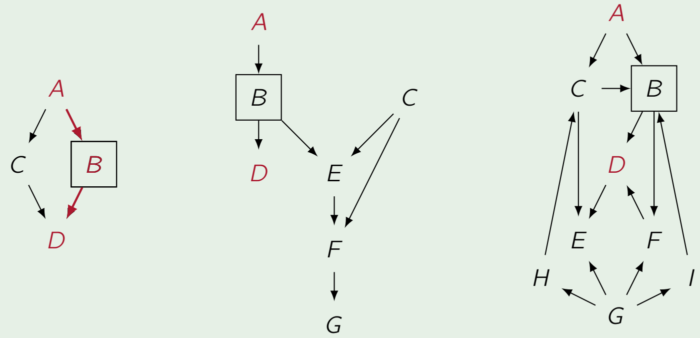

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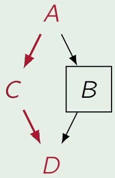

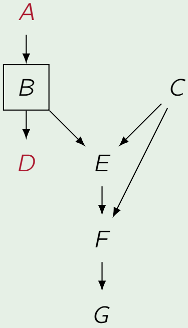

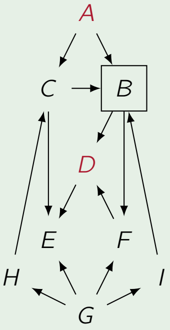

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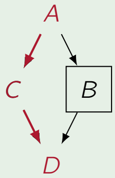

$< A, C, D >$ is not blocked

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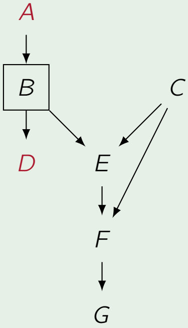

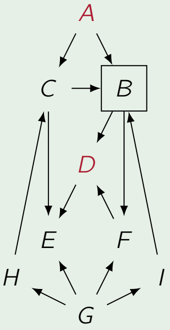

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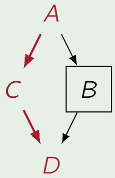

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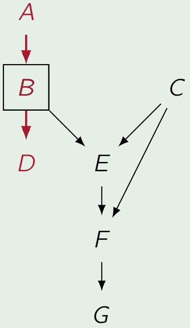

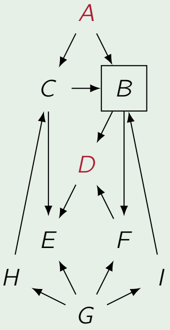

Reading conditional independencies using d-separation

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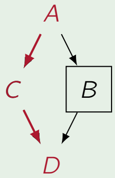

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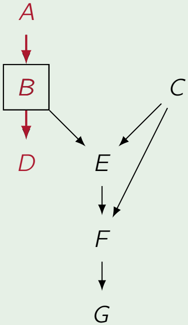
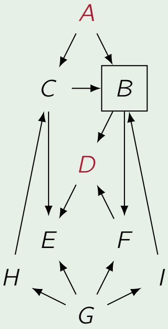


All paths are blocked

$\implies A \perp\!\!\!\perp_P D \mid B$



Reading conditional independencies using d-separation

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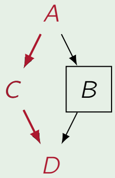

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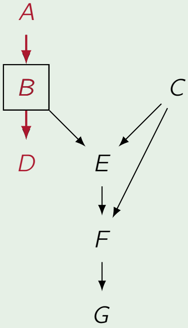

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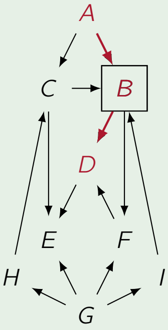

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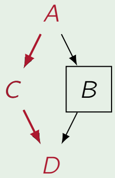

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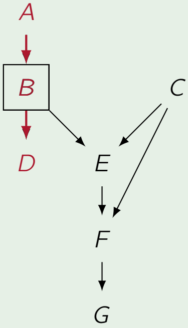

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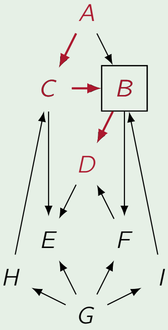

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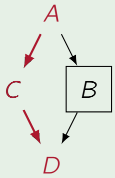

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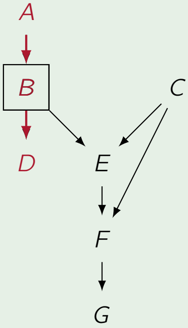

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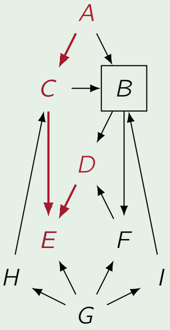

Reading conditional independencies using d-separation

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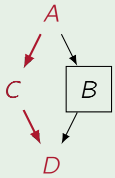

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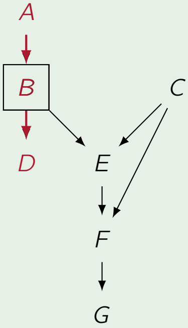

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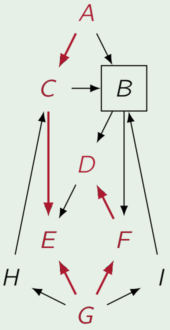

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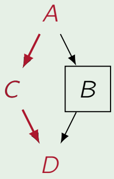

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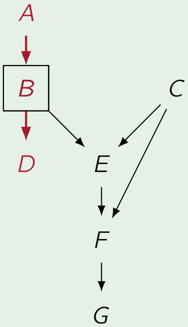

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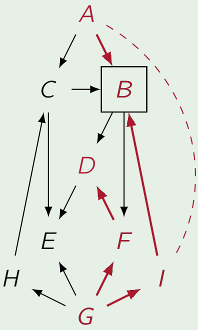

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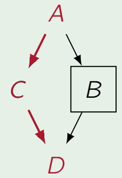

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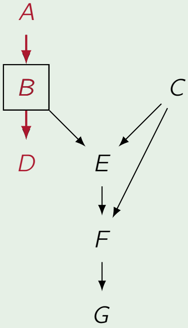

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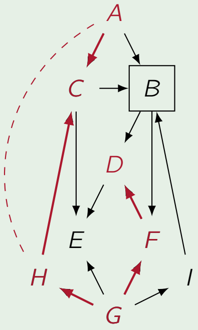

Reading conditional independencies using d-separation

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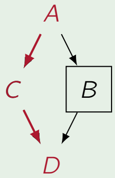

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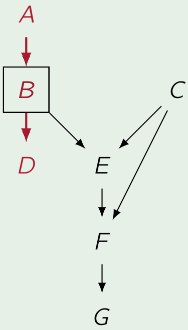

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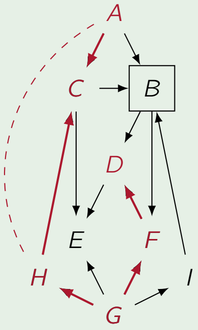


$< A, B, I, G, F, D >$ is not blocked

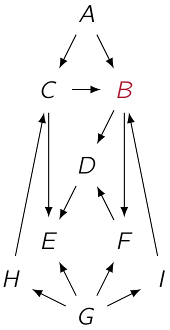
$\implies A \overset{?}{\perp\!\!\!\perp}_P D \mid B$

Python code: d-separation using pyCIPHOD package

```
#Load the pyCIPHOD package
from pyciphod.graphs import DirectedAcyclicGraph
from pyciphod.separation import d_separated
#Define the DAG
g = DirectedAcyclicGraph()
g.add_vertices(["A","B","C"])
g.add_directed_edge('B', 'A')
g.add_directed_edge('B', 'C')
#Check if B d-separates A and C
ds = g.d_separated(["A"], ["C"], ["B"])
print(ds)
```

Exercise 1

What are the relevant variables for predicting B?



3

Interventions

Causal effect

$$\text{The causal effect of } \mathbb{A} \text{ on } \mathbb{Y}$$
$$= \mathbb{E}(\mathbb{Y} \mid do(\mathbb{A} = \mathbb{a})) - \mathbb{E}(\mathbb{Y} \mid do(\mathbb{A} = \mathbb{a}')).$$

The operator $do()$ represents interventions.

Causal effect

The causal effect of $\mathbb{A}$ on $\mathbb{Y}$

$$= \mathbb{E}(\mathbb{Y} \mid do(\mathbb{A} = \mathbb{a})) - \mathbb{E}(\mathbb{Y} \mid do(\mathbb{A} = \mathbb{a}'))\text{.}$$

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Population

Causal effect

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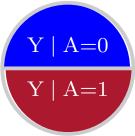
Population

Sub-populations



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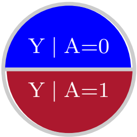
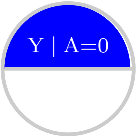
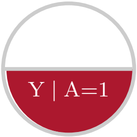
Population

Sub-populations



Conditioning





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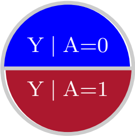
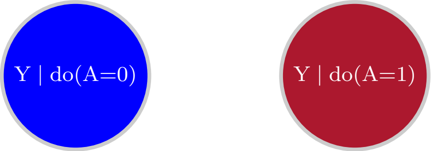
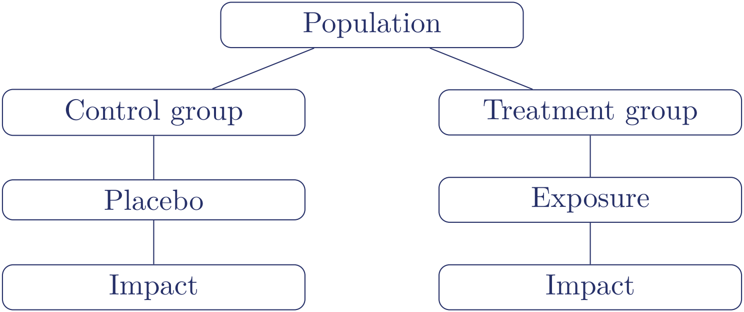


Conditioning

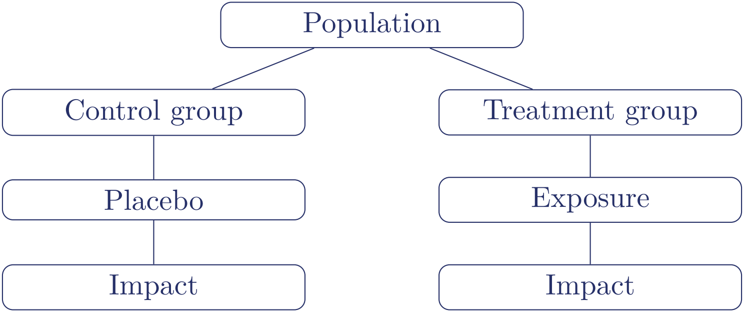
Intervening



Gold standard: Randomized controlled trials



Gold standard: Randomized controlled trials



Limitations:

- Unethical
- Infeasible
- Costly

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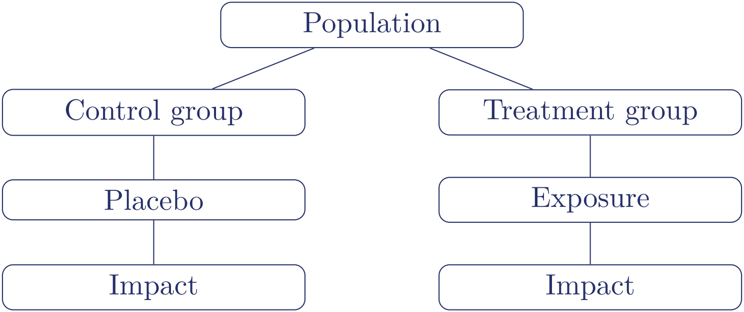


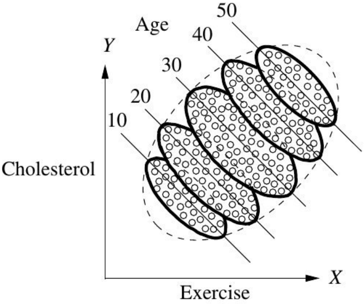
Limitations:

- Unethical
- Infeasible
- Costly

$\Longrightarrow$ Sometimes we have to rely on observational studies.

Simpson paradox

In a study, we measure weekly exercise and cholesterol levels across different age groups. What is the causal effect of exercise on cholesterol?



Causal inference in observational studies

Estimating causal effects from observational data necessarily requires strong assumptions.

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Two main frameworks

Potential outcomes

Structural causal models

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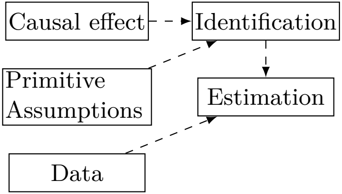
Two main frameworks

Potential outcomes



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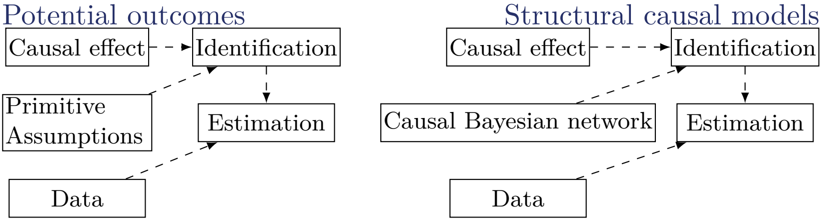

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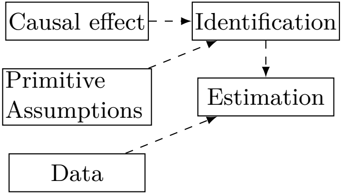

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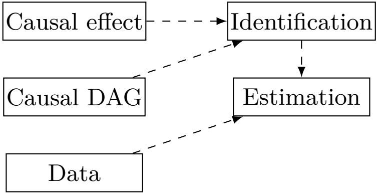

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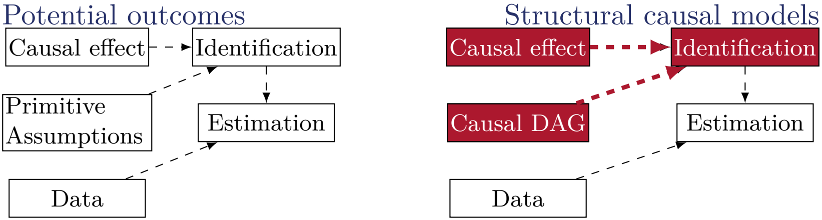

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Causal effect → Identification

Primitive Assumptions → Identification

Identification → Estimation

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Causal DAG → Identification

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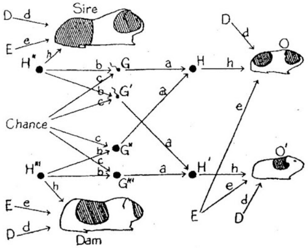

Sewal Wright

Causal inference in observational studies

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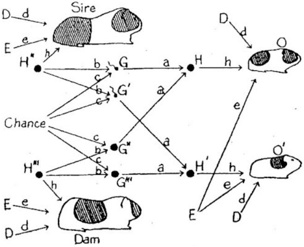

Sewal Wright

Judea Pearl

Causal DAGs

A causal DAG $\mathcal{G} = (\mathbb{V}, \mathbb{E})$ is a bayesian network where

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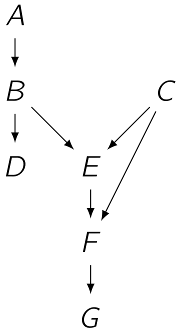

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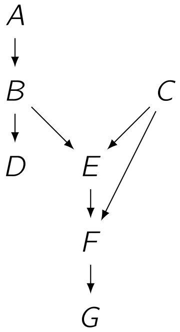

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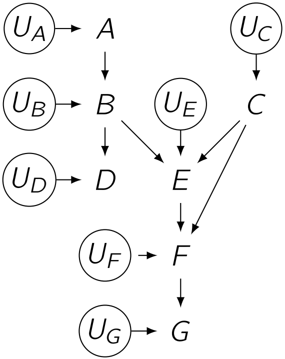

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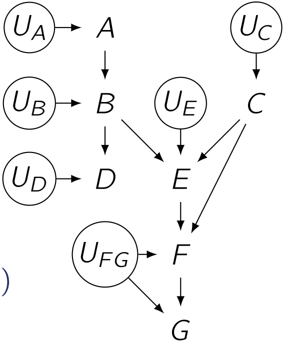

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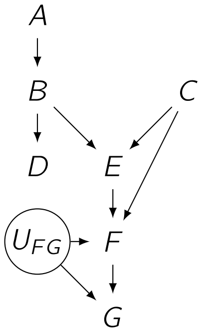

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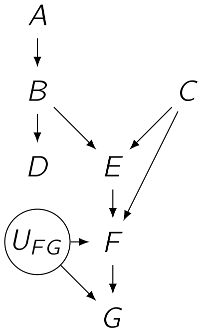

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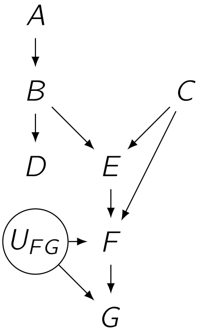

In a structural causal model, the intervention $\mathrm{do}(F = f)$ is defined by replacing the structural equation of F with $F := f$, leaving all other equations unchanged.

Bayesian networks vs causal DAGs

Not all Bayesian networks are necessarily causal DAGs.

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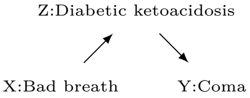

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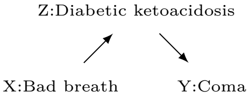

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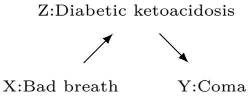

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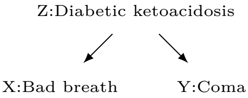

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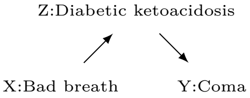

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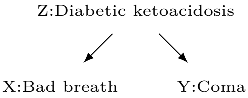
Oracle for conditional independencies



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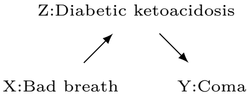

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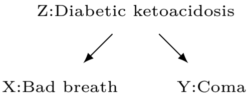

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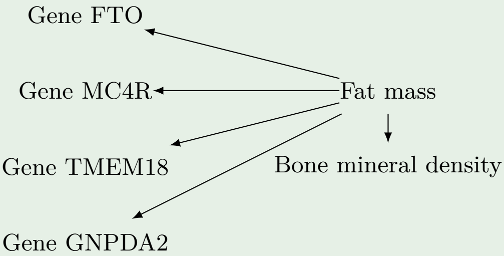
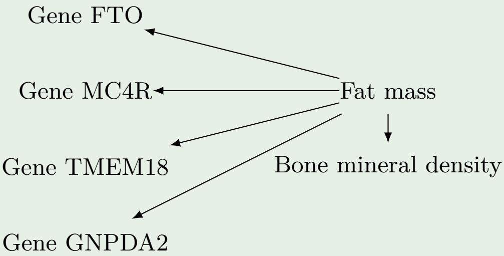
It is impossible to determine, without additional assumptions, which of the two DAGs is causal based solely on the observed distribution!

Example



Is this DAG causal?

Example



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Bayesian networks vs causal DAGs: examples

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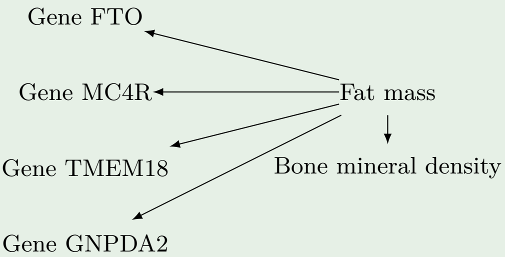

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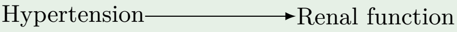

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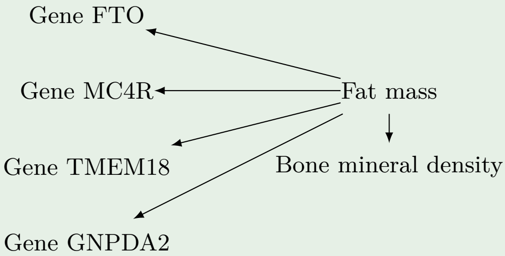

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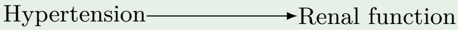

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In causal DAGs, the information lies in the absence of edges and the direction of arrows, rather than merely in the presence of edges.

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- When you are not sure if you need to add or not an edge (for example $Z \to A$) to the graph, ADD IT! (as long as you keep the graph acyclic)

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The causal effect of $\mathbb{A}$ on $\mathbb{Y} = \mathbb{E}(\mathbb{Y} \mid do(\mathbb{A} = \mathbb{a})) - \mathbb{E}(\mathbb{Y} \mid do(\mathbb{A} = \mathbb{a}'))$.

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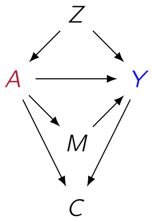

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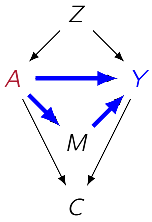

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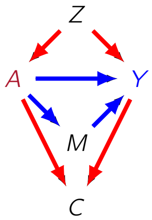

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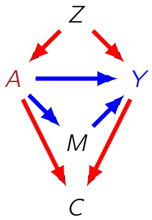

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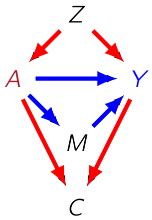

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In what follows, we will simplify the notation by using $P(\mathbb{y} \mid do(\mathbb{a}))$ instead of $P(\mathbb{Y} = \mathbb{y} \mid do(\mathbb{A} = \mathbb{a}))$.

Non-graphical solutions (1/2)

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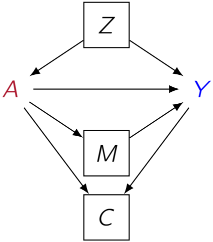

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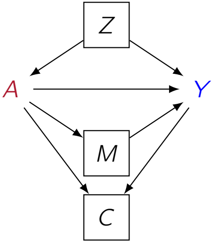

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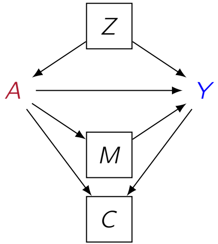

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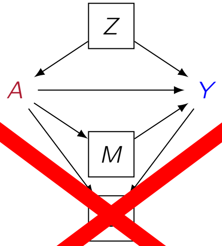

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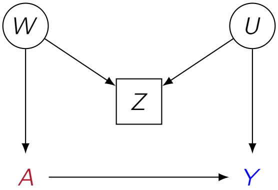

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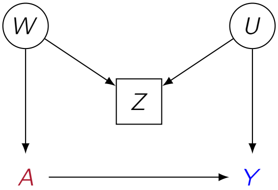

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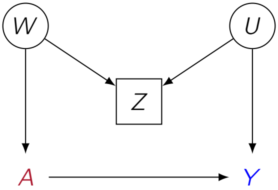

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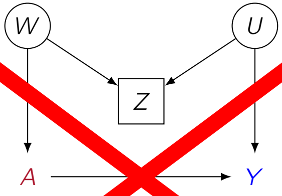

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The back-door criterion

A set of measured variables $\mathbb{Z}$ satisfies the back-door criterion relative to an ordered pair of variables (A, Y) in causal DAG $\mathcal{G}$ if:

- No node in $\mathbb{Z}$ is a descendant of A; and
- $\mathbb{Z}$ blocks all non-causal paths between A and Y.

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This criterion can be extended to a set $\mathbb{A}$ and a set $\mathbb{Y}$.

The back-door criterion: intuition

- Why "no node in $\mathbb{Z}$ is a descendant of A"?

- Why "$\mathbb{Z}$ blocks all non-causal paths between A and Y"?

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$$A \rightarrow \boxed{Z} \rightarrow Y$$

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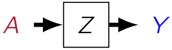

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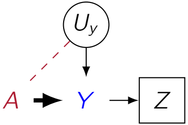

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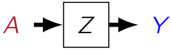

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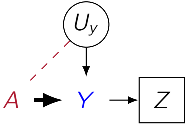

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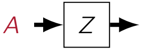

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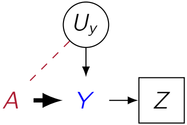

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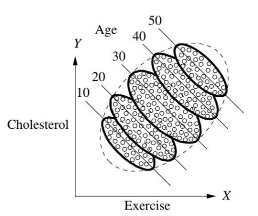
This one is the "obvious".

Simpson Paradox is no longer a paradox

Example

In a study, we measure weekly exercise and cholesterol levels across different age groups. What is the causal effect of exercise on cholesterol $P(c \mid do(e))$?



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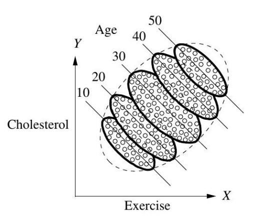

$$
\begin{tikzcd}
 & \text{Age} \arrow[dl] \arrow[dr] & \\
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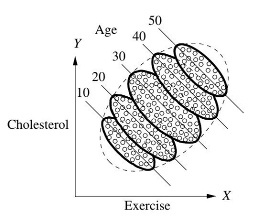

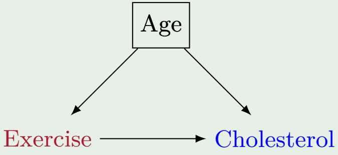

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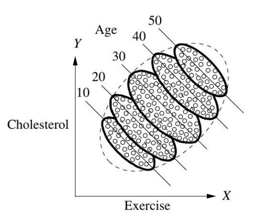

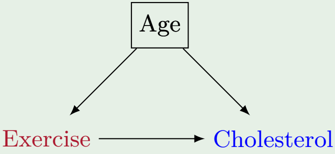

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In this study, we aim to estimate the effect of the neighborhood's socioeconomic status (N) on the reduction of walking within the neighborhood (R), $P(r \mid do(n))$?

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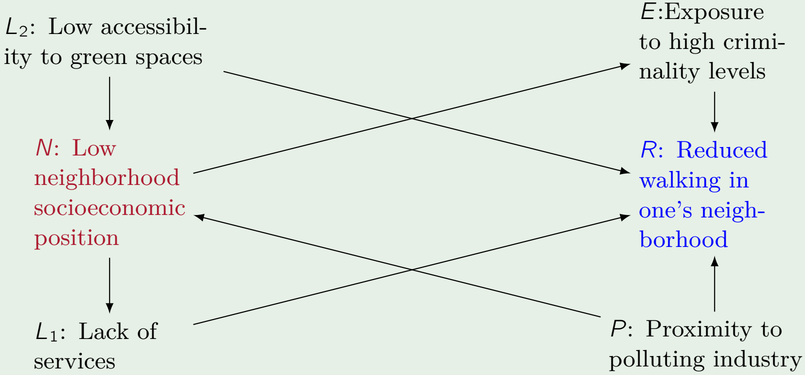

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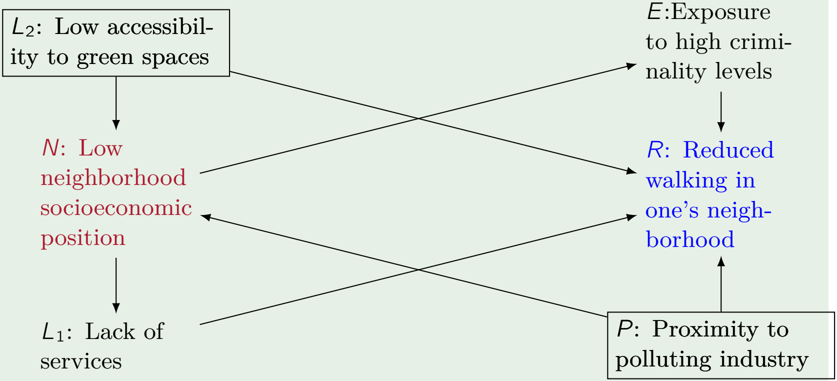

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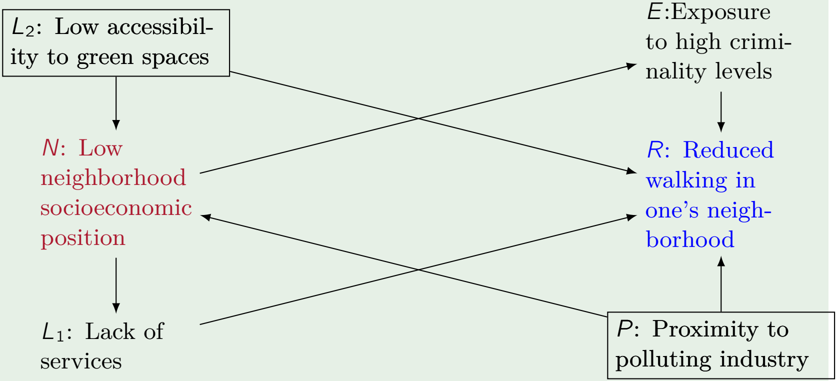

$$P(r \mid do(n)) = \sum P(r|n, l_2, p)P(l_2, p)$$

Back-door criterion in action 2

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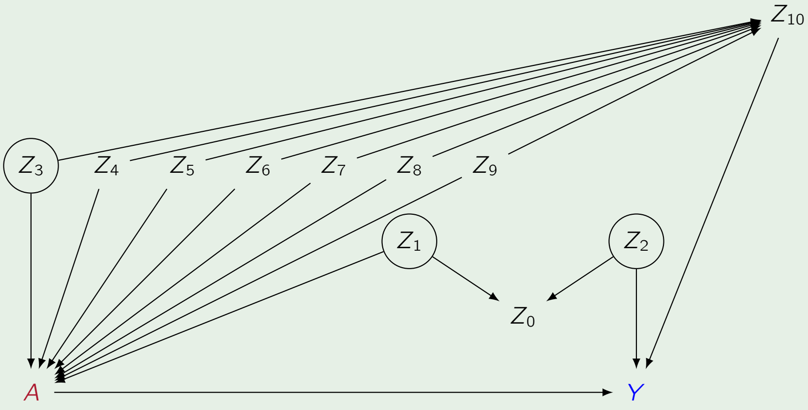
Which adjustment sets allow us to identify $\Pr(y \mid do(a))$?



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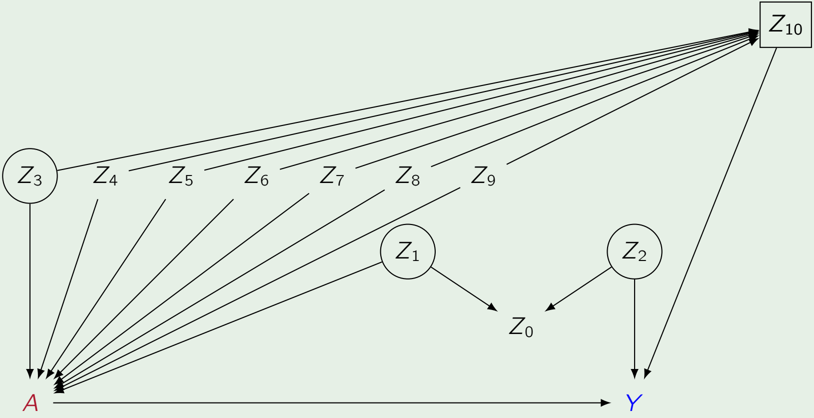

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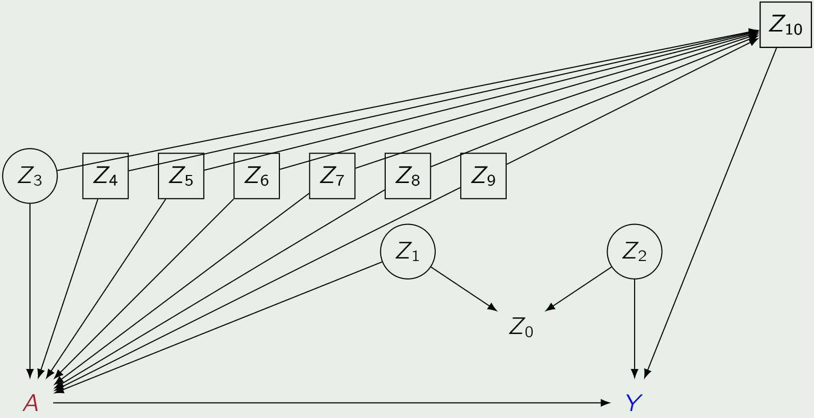

Finding the optimal set satisfying the back-door criterion

Many sets of variables can satisfy the back-door criterion:

- All these sets successfully remove bias.
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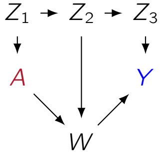

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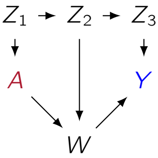

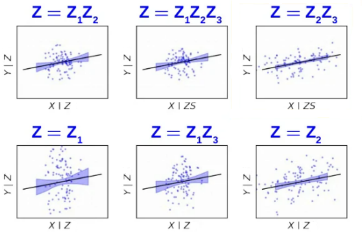

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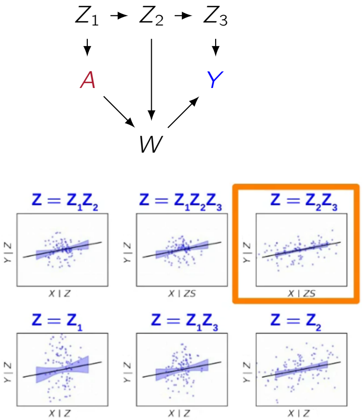

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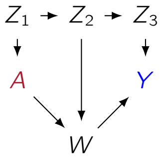

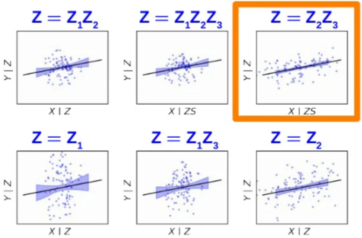

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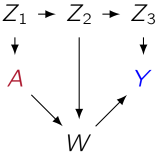

A set $\mathbb{Z}$ satisfying the back-door criterion relative to (A, Y) is optimal if:

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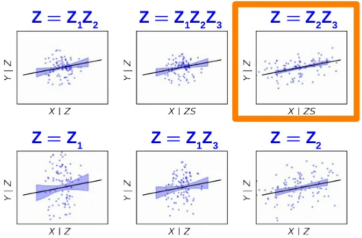

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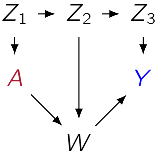

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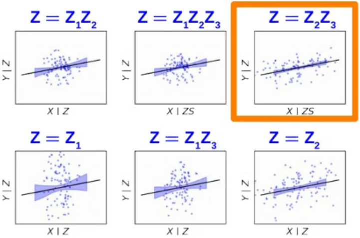
- $\mathbb{Z}$ includes all parents of any mediator between A and Y, that are not themselves mediators between A and Y.



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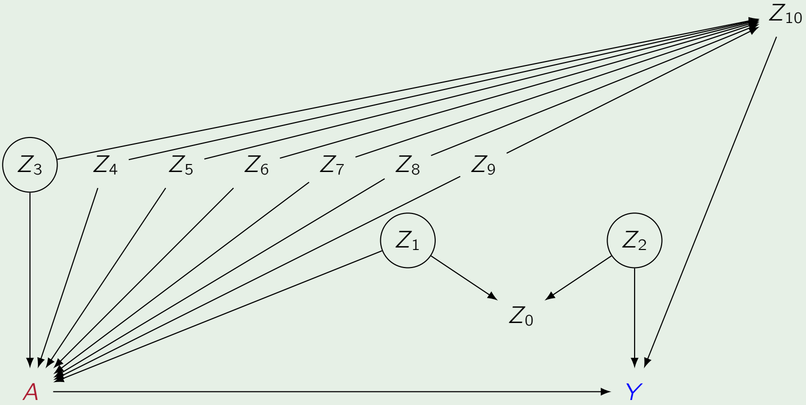

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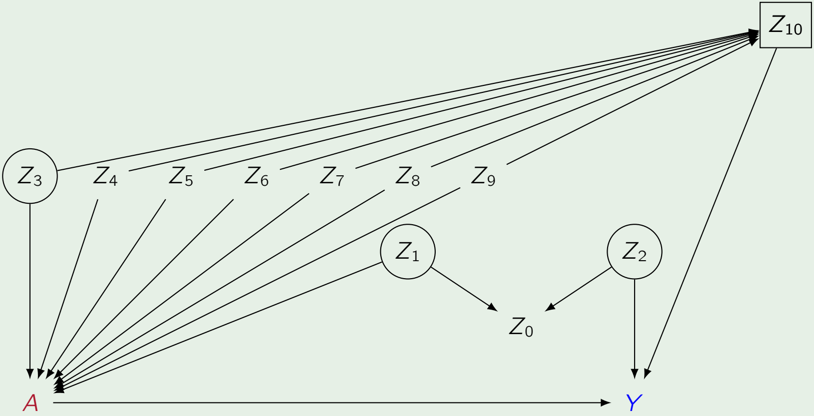

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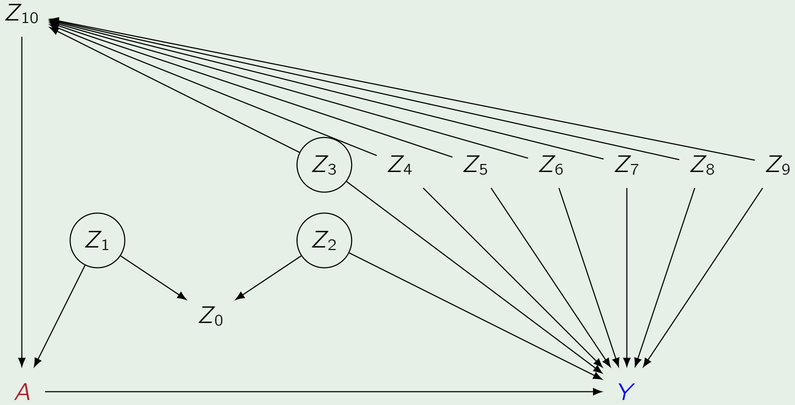

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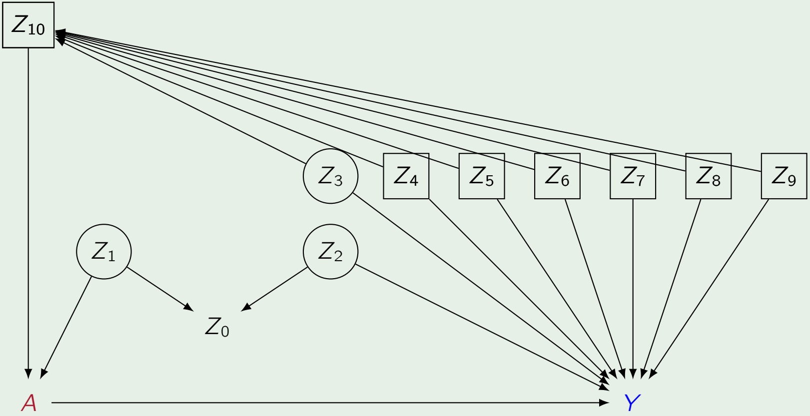

Optimal set in action 3

Example

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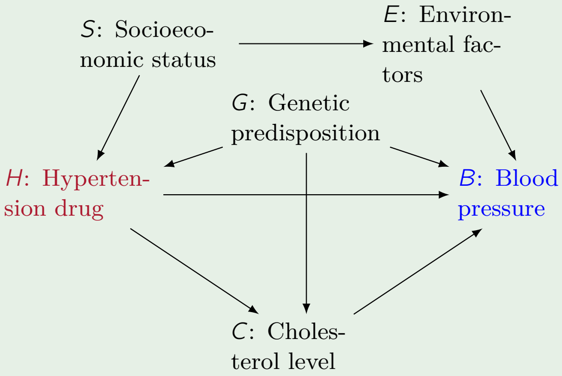

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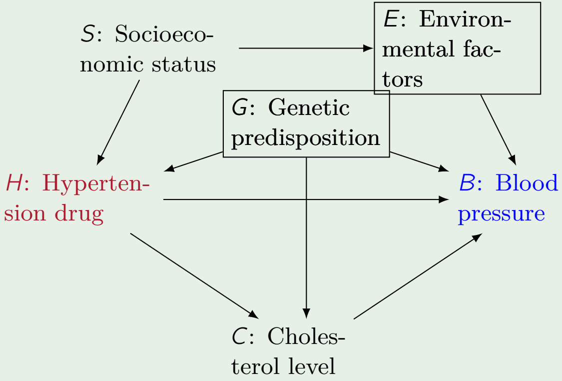

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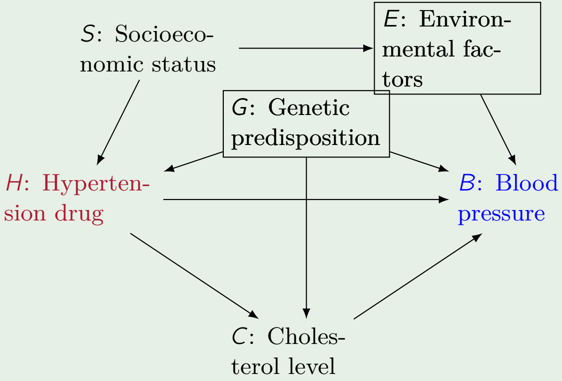

$$P(b \mid do(h)) = \sum_{g,e} P(b|h,g,e)P(g,e)$$

The back-door criterion is not complete:

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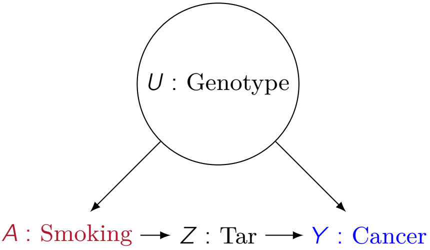
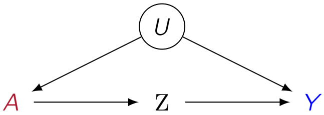
Going beyond the back-door




U
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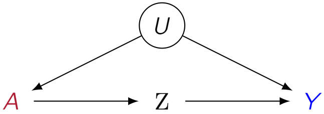

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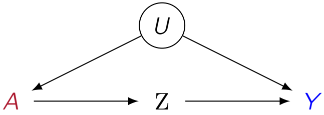

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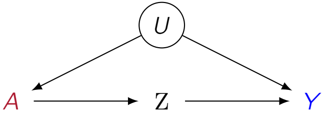

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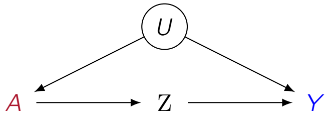

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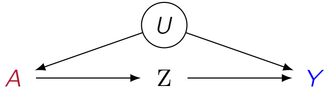
The front-door criterion



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- $\mathbb{Z}$ intercepts all causal paths from A to Y;
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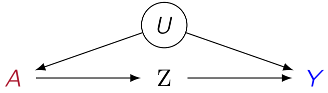

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Theorem

If $\mathbb{Z}$ satisfies the front-door criterion relative to (A, Y) and if $P(a, \mathbb{z}) > 0$, then the causal effect of A on Y is identifiable and is given by

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Back-door and Front-door criteria in a real application

In this study, Piccininni et al. were interested in estimating the effect of Mobile Stroke Unit dispatch (M) on functional outcomes (F). In other words, $P(f \mid do(m))$?

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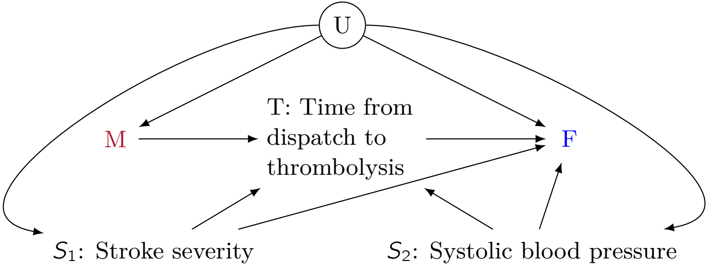

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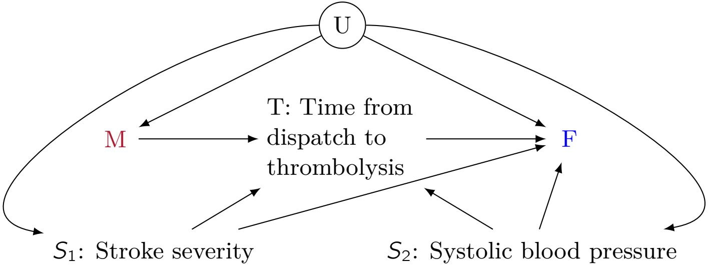

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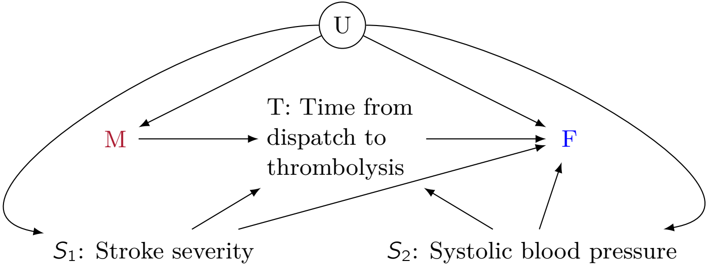

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Best paper in Epidemiology in 2024!

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The front-door criterion is not complete:

- If no set satisfies the front-door criterion for $P(y \mid do(x))$, this does not necessarily mean that $P(y \mid do(x))$ is not identifiable.

The combination of the back-door and the front-door criteria is also not complete.

The do-calculus

Mutilated graphs:

$$\begin{tikzcd}
B \arrow[r] \arrow[d] & Z \arrow[d] & C \arrow[l] \arrow[d] \\
D & A \arrow[l] \arrow[r] & E
\end{tikzcd}$$

$\mathcal{G}$

Mutilated graphs:

$$\begin{tikzcd} B \arrow[r] \arrow[d] & Z \arrow[d] & C \arrow[l] \arrow[d] \\ D & A \arrow[l] \arrow[r] & E \end{tikzcd} \qquad \begin{tikzcd} B \arrow[d] & Z \arrow[d] & C \arrow[d] \\ D & A \arrow[l] \arrow[r] & E \end{tikzcd}$$

$\mathcal{G}$ $\qquad\qquad$ $\mathcal{G}_{\overline{Z}}$

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$\mathcal{G}_{\overline{Z}}$

$$\begin{tikzcd} B \arrow[r] \arrow[d] & Z \arrow[d] & C \arrow[l] \arrow[d] \\ D & A & E \end{tikzcd}$$

$\mathcal{G}_{\underline{A}}$

The do-calculus

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$\mathcal{G}$

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$\mathcal{G}_{\overline{Z}}$

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The do-calclulus:

Rule 1: $P(\mathbb{y} \mid do(\mathbb{z}), \mathbb{a}, \mathbb{w}) = P(\mathbb{y} \mid do(\mathbb{z}), \mathbb{w})$ if $(\mathbb{Y} \perp\!\!\!\perp \mathbb{A} \mid \mathbb{Z}, \mathbb{W})_{\mathcal{G}_{\overline{\mathbb{Z}}}}$

Rule 2: $P(\mathbb{y} \mid do(\mathbb{z}, \mathbb{a}), \mathbb{w}) = P(\mathbb{y} \mid do(\mathbb{z}), \mathbb{a}, \mathbb{w})$ if $(\mathbb{Y} \perp\!\!\!\perp \mathbb{A} \mid \mathbb{Z}, \mathbb{W})_{\mathcal{G}_{\overline{\mathbb{Z}}\underline{\mathbb{A}}}}$

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The do-calculus

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$$\begin{tikzcd} B \arrow[r] \arrow[d] & Z \arrow[d] & C \arrow[l] \arrow[d] \\ D & A \arrow[l] \arrow[r] & E \end{tikzcd} \qquad \begin{tikzcd} B \arrow[d] & Z \arrow[d] & C \arrow[d] \\ D & A \arrow[l] \arrow[r] & E \end{tikzcd} \qquad \begin{tikzcd} B \arrow[r] \arrow[d] & Z \arrow[d] & C \arrow[l] \arrow[d] \\ D & A & E \end{tikzcd}$$

$$\mathcal{G} \qquad\qquad\qquad \mathcal{G}_{\overline{Z}} \qquad\qquad\qquad \mathcal{G}_{\underline{A}}$$

The do-calclulus:

Rule 1: $P(\mathbb{y} \mid \mathrm{do}(\mathbb{z}), \mathbb{a}, \mathbb{w}) = P(\mathbb{y} \mid \mathrm{do}(\mathbb{z}), \mathbb{w})$ if $(\mathbb{Y} \perp\!\!\!\perp \mathbb{A} \mid \mathbb{Z}, \mathbb{W})_{\mathcal{G}_{\overline{\mathbb{Z}}}}$

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Theorem

$P(\mathbb{y} \mid do(\mathbb{a}))$ is identifiable if and only if there exists a finite sequence of transformations, each conforming to either one of the Rules 1-3 or some standard probability manipulations, that reduces $P(\mathbb{y} \mid do(\mathbb{a}))$ into a do-free formula.

The do-calculus in action

Rule 1: $P(\mathbb{y} \mid \mathrm{do}(\mathbb{z}), \mathbb{x}, \mathbb{w}) = P(\mathbb{y} \mid \mathrm{do}(\mathbb{z}), \mathbb{w})$ if $(\mathbb{Y} \perp\!\!\!\perp \mathbb{X} \mid \mathbb{Z}, \mathbb{W})_{\mathcal{G}_{\overline{\mathbb{Z}}}}$

Rule 2: $P(\mathbb{y} \mid \mathrm{do}(\mathbb{z}, \mathbb{x}), \mathbb{w}) = P(\mathbb{y} \mid \mathrm{do}(\mathbb{z}), \mathbb{x}, \mathbb{w})$ if $(\mathbb{Y} \perp\!\!\!\perp \mathbb{X} \mid \mathbb{Z}, \mathbb{W})_{\mathcal{G}_{\overline{\mathbb{Z}}\underline{\mathbb{X}}}}$

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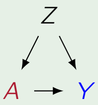
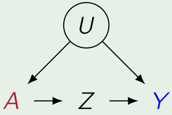
Example





The do-calculus in action

Rule 1: $P(\mathbb{y} \mid \mathrm{do}(\mathbb{z}), \mathbb{x}, \mathbb{w}) = P(\mathbb{y} \mid \mathrm{do}(\mathbb{z}), \mathbb{w})$ if $(\mathbb{Y} \perp\!\!\!\perp \mathbb{X} \mid \mathbb{Z}, \mathbb{W})_{\mathcal{G}_{\overline{\mathbb{Z}}}}$

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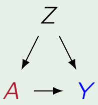
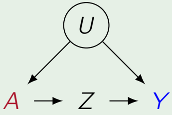
Example





$P(y \mid do(a))$

$= \sum_z P(\mathbb{y} \mid do(a), z) P(z \mid do(a))$

$= \sum_z P(y \mid a, z) P(z \mid do(a))$ (Rule 2)

$= \sum_z P(y \mid a, z) P(z)$ (Rule 3)

The do-calculus in action

Rule 1: $P(\mathbb{y} \mid \mathrm{do}(\mathbb{z}), \mathbb{x}, \mathbb{w}) = P(\mathbb{y} \mid \mathrm{do}(\mathbb{z}), \mathbb{w})$ if $(\mathbb{Y} \perp\!\!\!\perp \mathbb{X} \mid \mathbb{Z}, \mathbb{W})_{\mathcal{G}_{\overline{\mathbb{Z}}}}$

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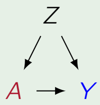
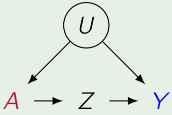
Example



$$
\begin{aligned}
&P(y \mid do(a)) \\
&= \sum_z P(\mathbb{y} \mid do(a), z) P(z \mid do(a)) \\
&= \sum_z P(y \mid a, z) P(z \mid do(a)) \quad \text{(Rule 2)} \\
&= \sum_z P(y \mid a, z) P(z) \quad \text{(Rule 3)}
\end{aligned}
$$

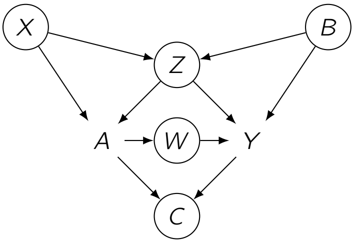


$$
\begin{aligned}
&P(y \mid do(a)) \\
&= \sum_z P(y \mid do(a), z) P(z \mid do(a)) \\
&= \sum_z P(y \mid do(a), do(z)) P(z \mid a) && \text{(Rule 2) x2} \\
&= \sum_z P(y \mid do(z)) P(z \mid a) && \text{(Rule 3)} \\
&= \sum_{z,a'} P(y \mid do(z), a') P(a' \mid do(z)) P(z \mid a) \\
&= \sum_{z,a'} P(y \mid z, a') P(a' \mid do(z)) P(z \mid a) && \text{(Rule 2)}
\end{aligned}
$$

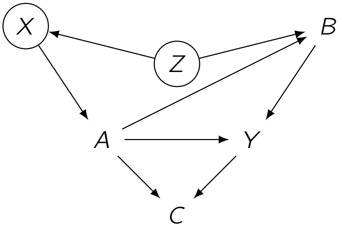
```
#Load the pyCIPHOD package
from pyciphod.graphs import DirectedAcyclicGraph
from pyciphod.causal_reasoning import back_door_criterio
from pyciphod.causal_reasoning import front_door_criteri
#Define the DAG
g = DirectedAcyclicGraph()
g.add_vertices(["A","B","C", "D"])
g.add_directed_edge('B', 'A')
g.add_directed_edge('B', 'C')
g.add_directed_edge('A', 'D')
g.add_directed_edge('D', 'C')
#Check if B satifies the back-door criterion for (A,C)
bd = back_door_criterion(g, ["A"], ["C"], ["B"])
print(bd)
#Check if D satifies the front-door criterion for (A,C)
fd = front_door_criterion(g, ["A"], ["C"], ["D"])
print(fd)
```

Exercise 2

Consider that in the following causal DAG, only A and Y, and one additional variable can be measured. Which variable would allow the identification of $P(y \mid do(a))$?



- Consider the following causal DAG. List all sets of variables that satisfy the back-door criterion for $P(y \mid do(a))$;



Exercise 4

Is $\{Z\}$ a good, bad or neutral adjustment set for $P(y \mid do(a))$?

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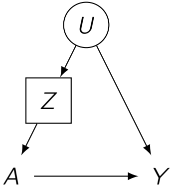

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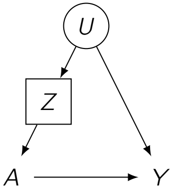
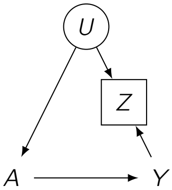


- Z blocks all non-causal paths and it is not a descendant of A
  $\implies \{Z\}$ is a good adjustment set.

Exercise 4

Is $\{Z\}$ a good, bad or neutral adjustment set for $P(y \mid do(a))$?



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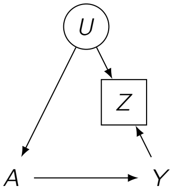

- Z activates a back-door path
  $\implies \{Z\}$ is a bad adjustment set.

Exercise 4

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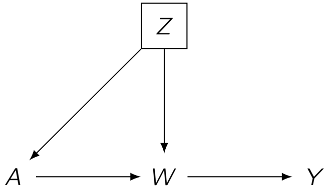

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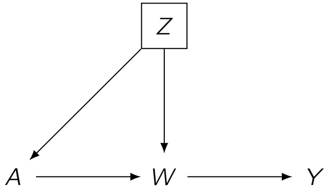

- Z blocks all non-causal paths and it is not a descendant of A
  $\implies$ $\{Z\}$ is a good adjustment set.

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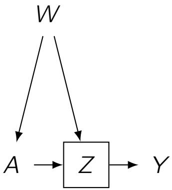
Is $\{Z\}$ a good, bad or neutral adjustment set for $P(y \mid do(a))$?



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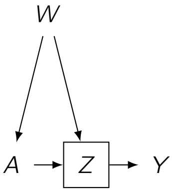

- Z d-separates A from Y
  $\implies$ $\{Z\}$ is a bad adjustment set.

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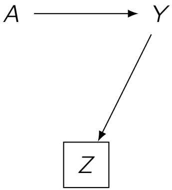


Exercise 4

Is $\{Z\}$ a good, bad or neutral adjustment set for $P(y \mid do(a))$?

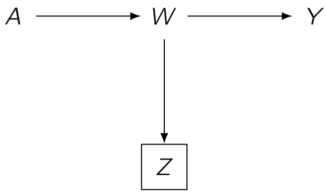
$$
\begin{tikzcd}
A \arrow[r] & Y \arrow[dl] \\
\boxed{Z} &
\end{tikzcd}
$$

- Selection bias

  $\Longrightarrow$ $\{Z\}$ is a bad control.

Exercise 4

Is $\{Z\}$ a good, bad or neutral adjustment set for $P(y \mid do(a))$?



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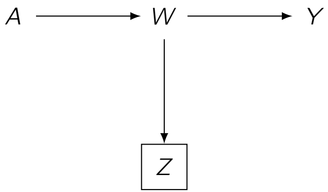

- Z is a descendant of A
  $\implies \{Z\}$ is a bad adjustment set.

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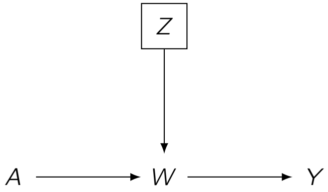

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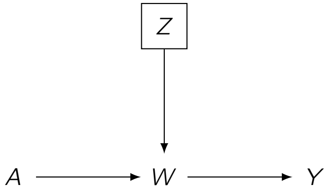

- Z is a parent of a mediator W
  $\implies$ $\{Z\}$ is a neutral adjustment set but good for optimality.

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4

Conclusion

Conclusion

Counterfactuals

Interventions

Associations

Counterfactuals

Interventions

Bayesian network

Associations

Counterfactuals

Causal DAG

Interventions

Bayesian network

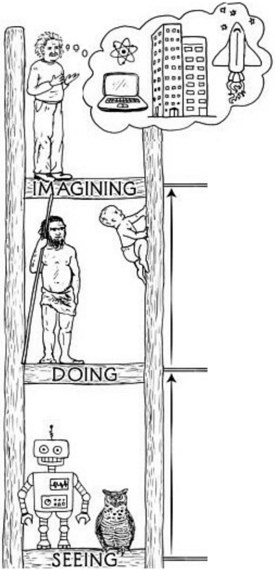
Associations

Conclusion

? | Counterfactuals

Causal DAG | Interventions

Bayesian network | Associations



5

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THANK YOU!

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