Causal discovery

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Santé Sorbonne Université

Sommaire

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Preliminaries

Reminder: Causal DAGs

A causal DAG $\mathcal{G} = (\mathbb{V}, \mathbb{E})$ is a directed acyclic graph where

- $\mathbb{V}$ is the set of vertices representing random variables.
- $\mathbb{E}$ is the set of directed edges representing causal relations between these variables.

Markov condition:

$$\forall Y \in \mathsf{V}, Y \perp\!\!\!\perp_{\mathcal{P}} \mathsf{Z} \mid \textit{Parents}(Y).$$

- $\mathsf{Z} : \mathsf{V} \setminus \{\textit{Descendants}(Y), \textit{Parents}(Y)\}$.
- $\perp\!\!\!\perp_{\mathcal{P}}$: Statistical independence.

Basic structures:

Fork	Chain	Collider
$A \longleftarrow C \longrightarrow B$	$A \longrightarrow C \longrightarrow B$	$A \longrightarrow C \longleftarrow B$

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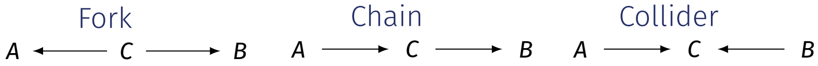
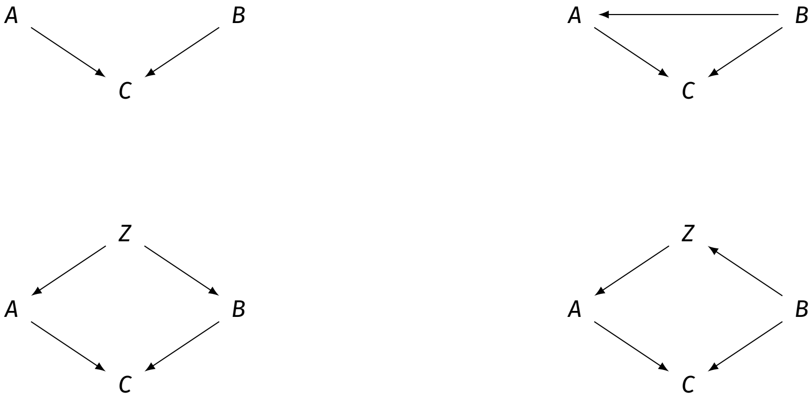


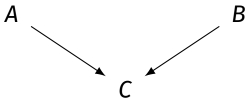
V-structure: collider where the extremities are not adjacent.

V-STRUCTURES

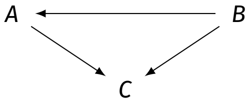


V-STRUCTURES

V-structure



Not v-structure



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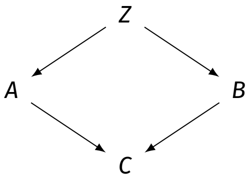

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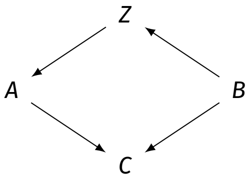


Reminder: Blocked path and d-separation

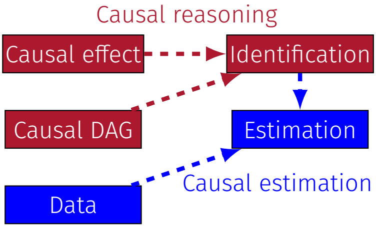
A path is said to be blocked by a set of vertices $\mathbb{Z} \in \mathbb{V}$ if:

- it contains a chain $A \to B \to C$ or a fork $A \leftarrow B \to C$ and $B \in \mathbb{Z}$; or
- it contains a collider $A \to B \leftarrow C$ such that no descendant of B is in $\mathbb{Z}$.

Given disjoint sets $\mathbb{X}, \mathbb{Y}, \mathbb{Z} \subseteq \mathbb{V}$, we say that $\mathbb{X}$ and $\mathbb{Y}$ are d-separated by $\mathbb{Z}$ if every path between a vertex in $\mathbb{X}$ and a vertex in $\mathbb{Y}$ is blocked by $\mathbb{Z}$ and we write $\mathbb{X} \perp\!\!\!\perp_{\mathcal{G}} \mathbb{Y} \mid \mathbb{Z}$.

$\mathbb{X} \perp\!\!\!\perp_{\mathcal{G}} \mathbb{Y} \mid \mathbb{Z} \Rightarrow \mathbb{X} \perp\!\!\!\perp_{P} \mathbb{Y} \mid \mathbb{Z}$
but $\mathbb{X} \not\!\perp\!\!\!\perp_{\mathcal{G}} \mathbb{Y} \mid \mathbb{Z} \not\Rightarrow \mathbb{X} \not\!\perp\!\!\!\perp_{P} \mathbb{Y} \mid \mathbb{Z}$

What if the causal DAG is unknown?



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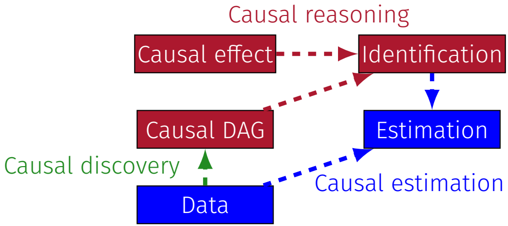

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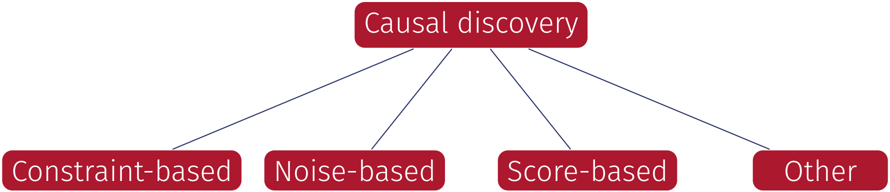
Constraint-based causal discovery

Families of causal discovery methods



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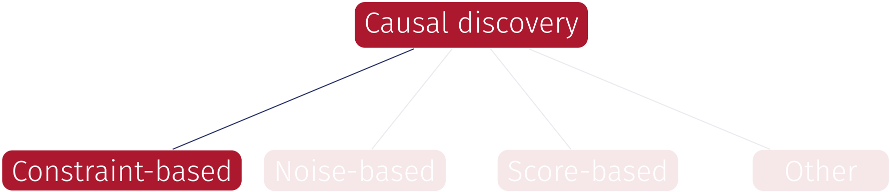

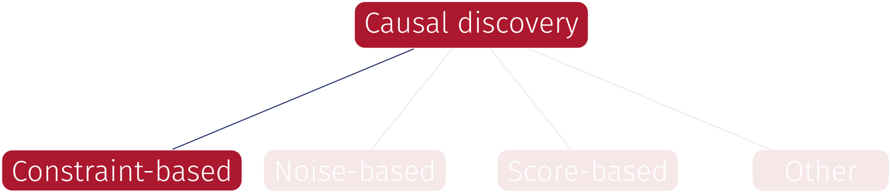

Constraint-based: run local tests of independence to create constraints on space of possible graphs.

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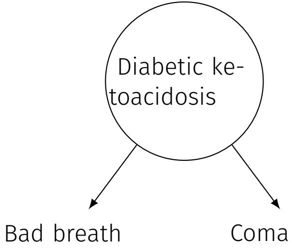

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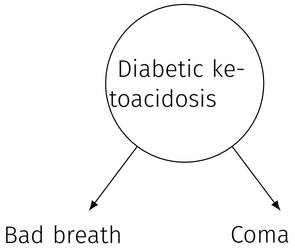
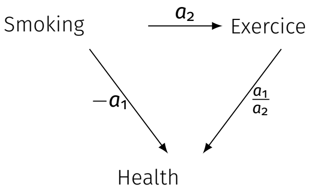
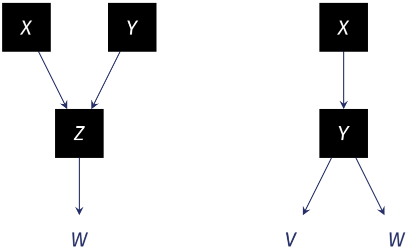




Deterministic relationships



(a) (b)

- (a) $Z = X + Y$:
- (b) $Y = 2X$:

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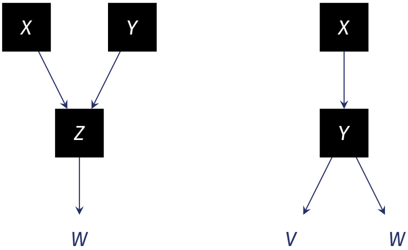

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- (a) $Z = X + Y$: $W \perp\!\!\!\perp_P Z \mid \{X, Y\}$, but $W \not\perp\!\!\!\perp_{\mathcal{G}} Z \mid \{X, Y\}$
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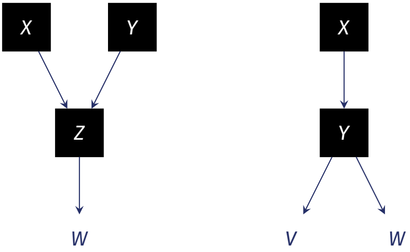

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Additional assumptions

Causal sufficiency: No hidden confounding.

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Markov equivalence class [12]

Equivalence in terms of conditional independence

$X \rightarrow Y$	$X \leftarrow Y$	$X \rightarrow Z \leftarrow Y$
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- All equivalent graphs can be represented by a completed partially DAG (CPDAG)
- This CPDAG is called the representative of the Markov equivalence class

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Finding skeleton and v-structures

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If $P(\mathbb{V})$ is faithful to some causal DAG $\mathcal{G}$ with vertex $\mathbb{V}$ then:

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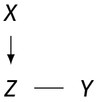
- Point 1 can be used to discover the skeleton of $\mathcal{G}$ from $P(\mathbb{V})$;
- Given the skeleton of $\mathcal{G}$, point 2 can be used to find all v-structures.

Meek orientation rules [6]

Suppose we already found the skeleton and all v-structures:

Meek-Rule 1:



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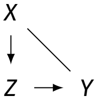

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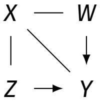

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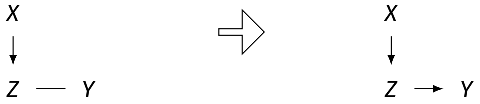

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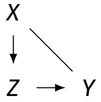

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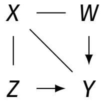

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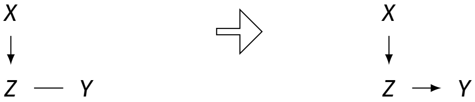

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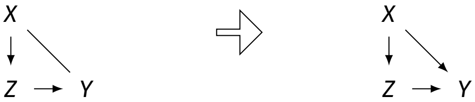

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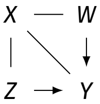

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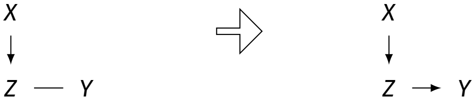

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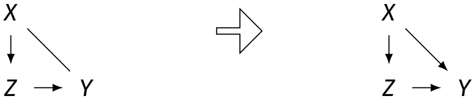

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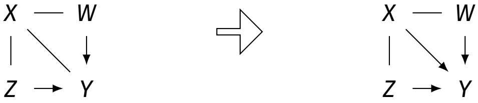

The PC algorithm [11]

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Theorem ([11])

Assume the distribution P is compatible and faithful to some causal DAG $\mathcal{G}$ and assume that we are given perfect conditional independence information about all pairs of variables. The PC algorithm returns the CPDAG of $\mathcal{G}$.

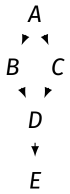
The optimal strategy of the PC algorithm [11]

Pruning unnecessary edges in optimal way:

- use a sequential conditioning strategy, where the size of the conditioning set increases progressively from 1 to p-2
- the conditioned set is the subset of the set of variables adjacent to tested variables
- storing the separation sets, which can later be used for orienting v-structures and other triplets

PC in action

- Suppose the causal DAG on the right
- Input: Observational data
- Output: CPDAG
- Assumptions: causal sufficiency, faithfulness



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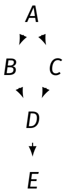

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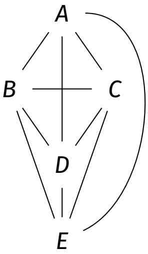

$card = 0$

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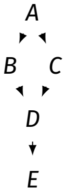

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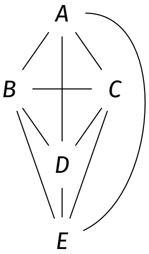

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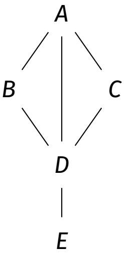

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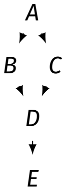

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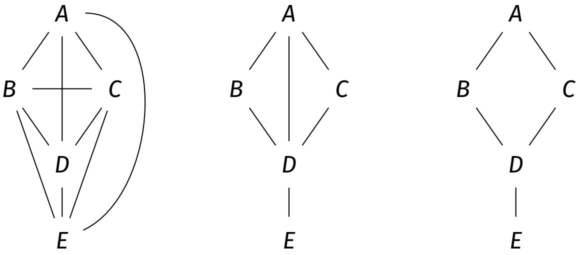

card = 0 *card* = 1 *card* = 2

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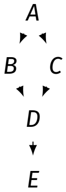

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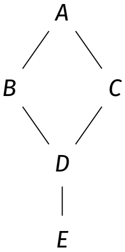

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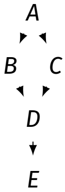
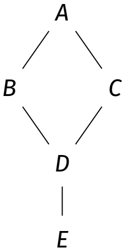


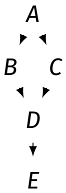
Orientation:



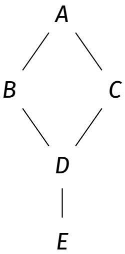
- Finding V-structures

PC in action

- Suppose the causal DAG on the right
- Input: Observational data
- Output: CPDAG
- Assumptions: causal sufficiency, faithfulness



Orientation:



- Finding V-structures
  - $B \perp\!\!\!\perp_P C \mid A \implies B \rightarrow D \leftarrow C$

PC in action

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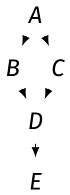

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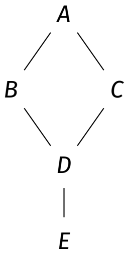

- Finding V-structures
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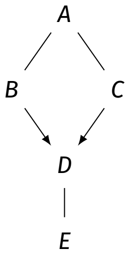

PC in action

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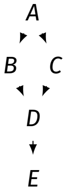

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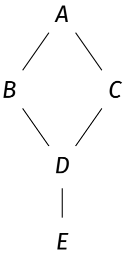
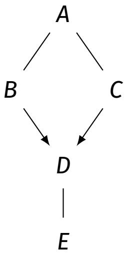
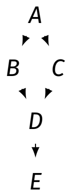


- Finding V-structures
  - $B \perp\!\!\!\perp_P C \mid A \implies B \to D \leftarrow C$

- Meek-Rule 1



- Suppose the causal DAG on the right
- Input: Observational data
- Output: CPDAG
- Assumptions: causal sufficiency, faithfulness



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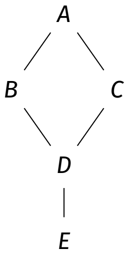
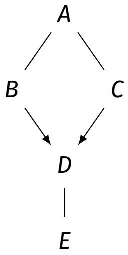


- Finding V-structures
  - $B \perp\!\!\!\perp_P C \mid A \implies B \rightarrow D \leftarrow C$
- Meek-Rule 1
  - $B \rightarrow D \;\&\; D - E \implies D \rightarrow E$



PC in action

- Suppose the causal DAG on the right
- Input: Observational data
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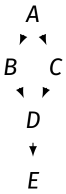
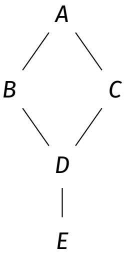


Orientation:



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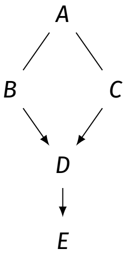


Incorporating background knowledge

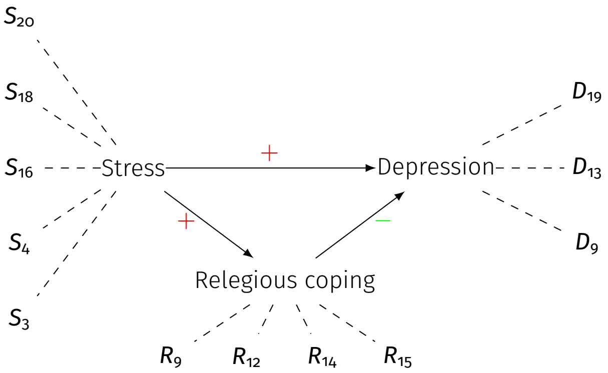
The PC algorithm can effectively incorporate background knowledge in the form of:

- Forbidden edges
- Required edges
- Forbidden orientations
- Required orientations

Real application of PC [9]

MSW students ($N = 127$); 61 item survey (Likert Scale)

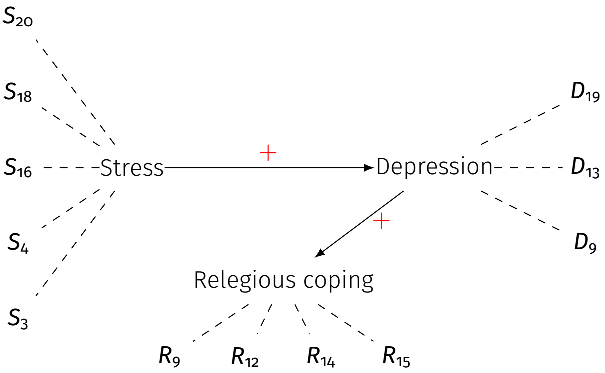
Specified graph



Real application of PC [9]

MSW students ($N = 127$); 61 item survey (Likert Scale)

Inferred graph (assuming stress is temporally prior)



Temporal PC

Main idea: Assign each variable X to an ordered tier (e.g., its measurement period).

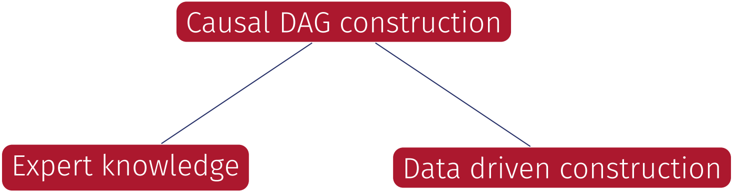
Tiered background information can be used in the PC algorithm in the following way:

- Skeleton phase: test $X \perp\!\!\!\perp Y \mid \mathbb{S}$ only if $\mathbb{S}$ is not later than both X and Y
- Orientation phase: if X is temporally prior to Y orient $X \rightarrow Y$

Temporal PC(tPC) [11, 8]: The output of the tPC is more informative then CPDAG, as temporal knowledge can orient edges that remain undirected in the CPDAG.

Life course study: Depression and heart disease etiology in the Metropolit cohort

Aim: Life course study on etiology of depression and heart disease in early old age



Life course study: Depression and heart disease etiology in the Metropolit cohort

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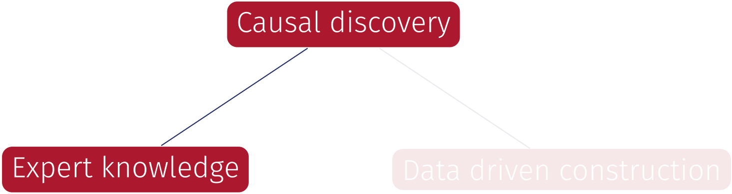
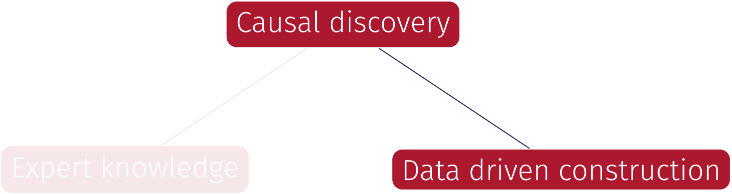


Two experts build a causal DAG (individual + consensus)

Life course study: Depression and heart disease etiology in the Metropolit cohort

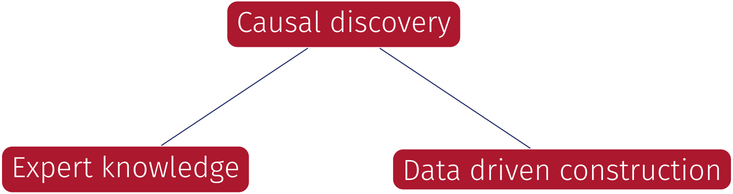
Aim: Life course study on etiology of depression and heart disease in early old age



Apply temporal PC algorithm on data

Life course study: Depression and heart disease etiology in the Metropolit cohort

Aim: Life course study on etiology of depression and heart disease in early old age



Compare two models

Life course study: Depression and heart disease etiology in the Metropolit cohort.

low paternal social class$_{\text{childhood}}$

$\text{retirement}_{\text{early old age}}$ maternal smoking$_{\text{childhood}}$ $\text{depression}_{\text{early old age}}$

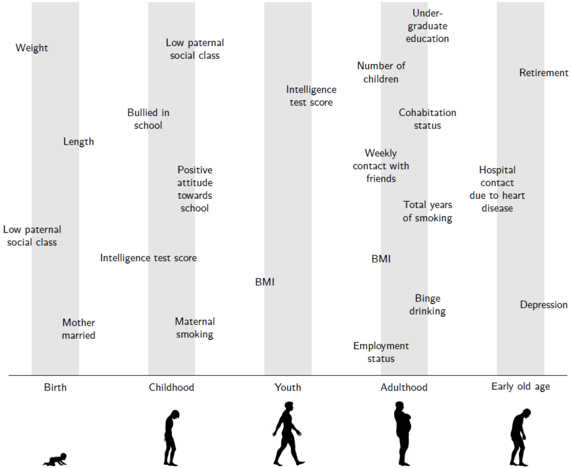
$\text{weight}_{\text{birth}}$ $\text{BMI}_{\text{youth}}$

$\text{BMI}_{\text{adult}}$

$\text{length}_{\text{birth}}$ bullied in school$_{\text{childhood}}$

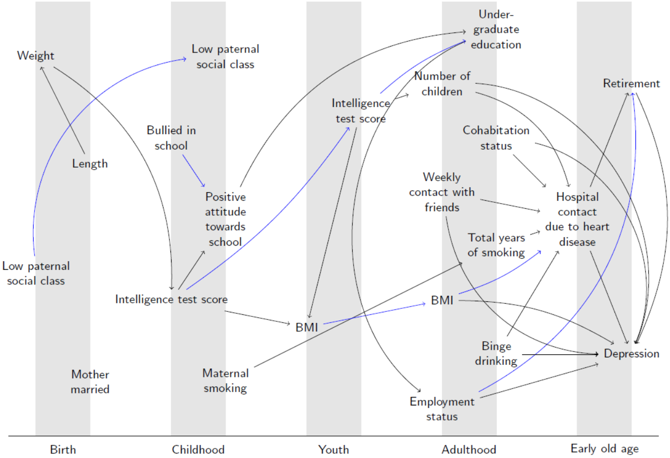
intelligence test score$_{\text{childhood}}$

Life course study: Depression and heart disease etiology in the Metropolit cohort.

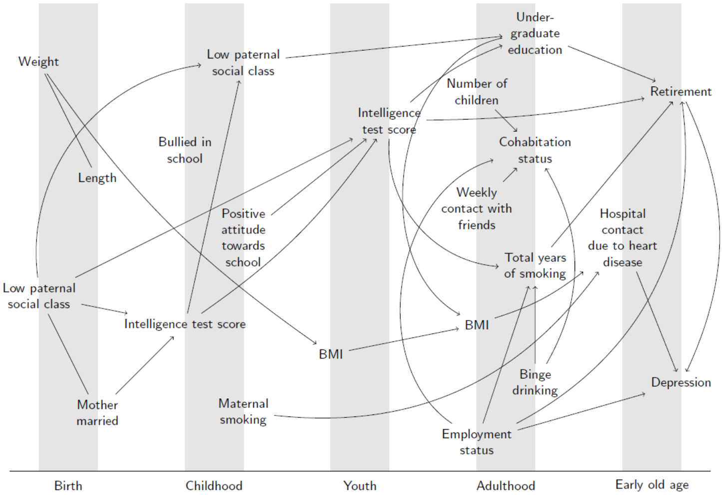


Petersen, Ekstrøm, Spirtes & Osler (2023). Constructing causal life course models: Comparative study of data-driven and theory-driven approaches. American Journal of Epidemiology

Metropolit results: Expert model



Metropolit results: Causal discovery model



Experts vs causal discovery

Shared adjacencies: no orientation disagreement, except one non-oriented edge.

		Experts Adjacency	Experts Non-adjacency
Causal discovery	Adjacency	**10**	20
	Non-adjacency	20	**181**

- **Shared adjacencies**: no orientation disagreement
- **Experts high confidence edges**: 6 out of 7 found by causal discovery algorithm
- 'Extra' 20 edges found by TPC
  - Low plausibility 3
  - Moderate plausibility 6
  - High plausibility 11

CPDAG: Can we use a backdoor?

Consider a causal DAG $\mathcal{G}$ and a causal effect $P(y \mid do(x))$. A set of variables $\mathbb{Z}$ satisfies the back-door criterion if:

- no vertex in $\mathbb{Z}$ is a descendant of X;
- $\mathbb{Z}$ **blocks** every non causal path from X to Y.

Works for DAGs, but we have CPDAG?

J. Pearl, M. Glymour, and N. P. Jewell. *Causation Inference In Statistics: A Primer*. Wiley, 2016.

Possible causal path [7]

Causal path: a directed path from X to Y, where all arrowheads point away from X. e.g. $X \to Z \to Y$

Possible causal path: a path from X to Y, such that there is no arrow pointing towards X e.g. $X - Z \to Y$

Non-causal path: a path from X to Y that is not possible causal, e.g. $X - Z \leftarrow Y$, $X \leftarrow Z - Y$

Possible descendants

If there is a possibly causal path from X to Y, then

- Y a **possible descendant** of X.

Simple extension of back-door criterion [7]

Consider a CPDAG $\mathcal{G}$ and a causal effect $P(y \mid do(x))$. A set of variables $\mathbb{Z}$ satisfies the genralized back-door criterion if:

- $\mathbb{Z}$ does not contain **possible** descendants of X in $\mathcal{G}$;
- $\mathbb{Z}$ **blocks** all non-causal paths from X to Y in $\mathcal{G}$

Simple extension of back-door criterion [7]

Consider a CPDAG $\mathcal{G}$ and a causal effect $P(y \mid do(x))$. A set of variables $\mathbb{Z}$ satisfies the genralized back-door criterion if:

- for any possible causal path between X and Y, the edge adjacent to X on the path is oriented;
- $\mathbb{Z}$ does not contain **possible** descendants of X in $\mathcal{G}$;
- $\mathbb{Z}$ **blocks** all non-causal paths from X to Y in $\mathcal{G}$

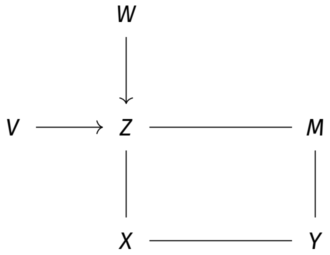
Simple extension of back-door criterion [7]

Consider a CPDAG $\mathcal{G}$ and a causal effect $P(y \mid do(x))$. A set of variables $\mathbb{Z}$ satisfies the genralized back-door criterion if:

- for any possible causal path between X and Y, the edge adjacent to X on the path is oriented;
- $\mathbb{Z}$ does not contain **possible** descendants of X in $\mathcal{G}$;
- $\mathbb{Z}$ **blocks** all non-causal paths from X to Y in $\mathcal{G}$

$$P(y \mid do(x)) = \sum_z P(y \mid x, z)P(z).$$

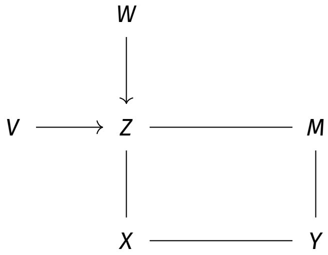
Example 1



Is it a CPDAG?

Is the generalized back-door criterion satisfied?

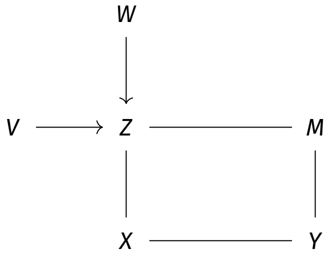
Example 1



Is it a CPDAG? No!

Is the generalized back-door criterion satisfied?

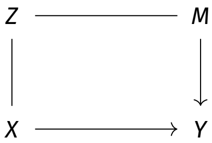
Example 1



Is it a CPDAG? No!

Is the generalized back-door criterion satisfied? No!

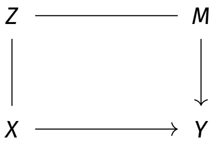
Example 2



Is it a CPDAG?

Is the generalized back-door criterion satisfied?

Example 2



Is it a CPDAG? Yes!

Is the generalized back-door criterion satisfied?

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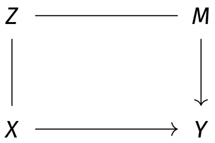
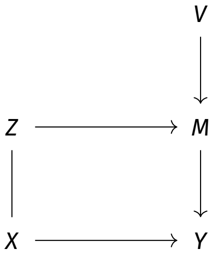


Is it a CPDAG? Yes!

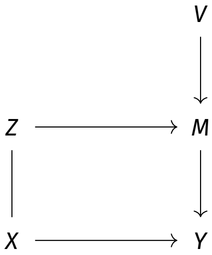
Is the generalized back-door criterion satisfied? No!
$X - Z - M \rightarrow Y$ is a possibly causal path and the first edge on the path is not oriented.

Example 3



Is it a CPDAG?
Is the generalized back-door criterion satisfied?

Example 3



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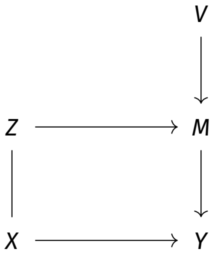

Is it a CPDAG? Yes!
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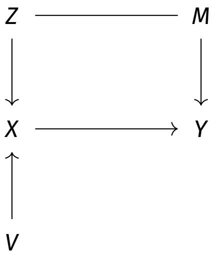
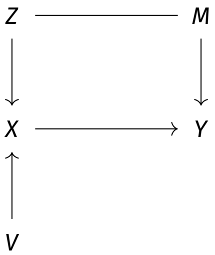
Example 4



Is it a CPDAG?
Is the generalization of the back-door criterion satisfied?

Example 4



Is it a CPDAG? Yes!
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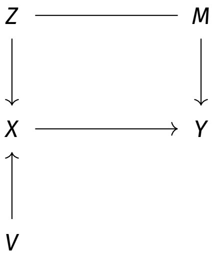
Example 4

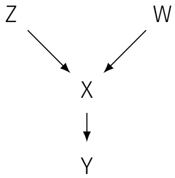


Is it a CPDAG? Yes!

Is the generalization of the back-door criterion satisfied? Yes!

$\{Z\}$, $\{M\}$, $\{Z, M\}$ satisfy the criterion.

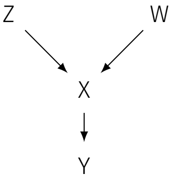
Beyond causal sufficiency (1/2)



Y-structure

$$Z \perp\!\!\!\perp_P W$$
$$Z \not\!\perp\!\!\!\perp_P W \mid X$$
$$Y \not\!\perp\!\!\!\perp_P Z$$
$$Y \perp\!\!\!\perp_P Z \mid X$$
$$Y \not\!\perp\!\!\!\perp_P W$$
$$Y \perp\!\!\!\perp_P W \mid X$$

Beyond causal sufficiency (1/2)



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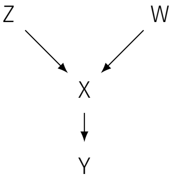
$$Y \not\perp\!\!\!\perp_P W$$
$$Y \perp\!\!\!\perp_P W \mid X$$

Can we have a hidden confounder between X and Y?

Beyond causal sufficiency (1/2)



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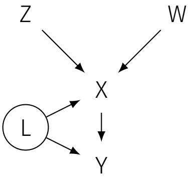

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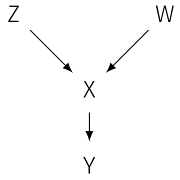
Beyond causal sufficiency (1/2)



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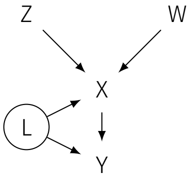

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$Y \not\perp\!\!\!\perp_P Z$

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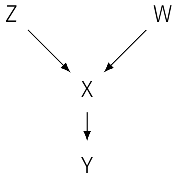
Beyond causal sufficiency (1/2)



Y-structure

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$$Z \not\perp\!\!\!\perp_P W \mid X$$
$$Y \not\perp\!\!\!\perp_P Z$$
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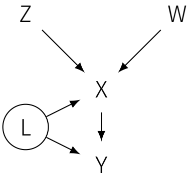
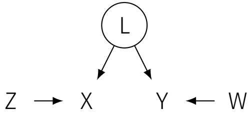


$$Z \perp\!\!\!\perp_P W$$
$$Z \not\perp\!\!\!\perp_P W \mid X$$
$$Y \not\perp\!\!\!\perp_P Z$$
$$Y \not\perp\!\!\!\perp_P Z \mid X$$
$$Y \not\perp\!\!\!\perp_P W$$
$$Y \not\perp\!\!\!\perp_P W \mid X$$

Pattern of independence can rule out hidden confounding.

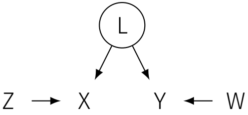
Beyond causal sufficiency (2/2)



W-structure

$$Z \not\perp\!\!\!\perp_P X$$
$$X \not\perp\!\!\!\perp_P Y$$
$$Y \not\perp\!\!\!\perp_P W$$
$$Z \perp\!\!\!\perp_P W$$
$$Z \perp\!\!\!\perp_P Y$$
$$X \perp\!\!\!\perp_P W$$
$$Z \not\perp\!\!\!\perp_P Y \mid X$$
$$X \not\perp\!\!\!\perp_P W \mid Y$$

Beyond causal sufficiency (2/2)



W-structure

$$Z \not\perp\!\!\!\perp_P X$$
$$X \not\perp\!\!\!\perp_P Y$$
$$Y \not\perp\!\!\!\perp_P W$$
$$Z \perp\!\!\!\perp_P W$$
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$$X \perp\!\!\!\perp_P W$$
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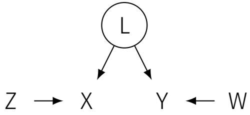

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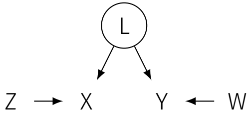

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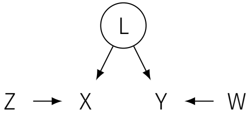

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Pattern of independence can suggest hidden confounding.

A glimpse of the FCI algorithm

The FCI algorithm extends the PC algorithm to accommodate hidden confounders:

A glimpse of the FCI algorithm

The FCI algorithm extends the PC algorithm to accommodate hidden confounders:

- Skeleton construction is much more complicated;
- Orientation is done using 10 different rules.

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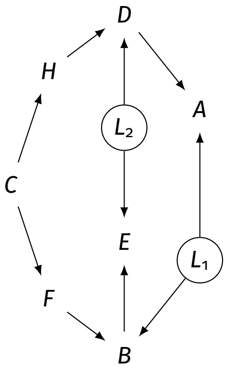

True graph

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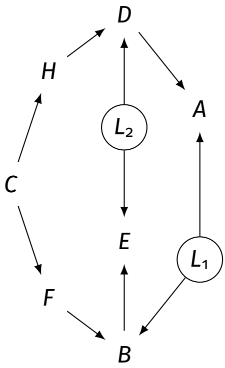
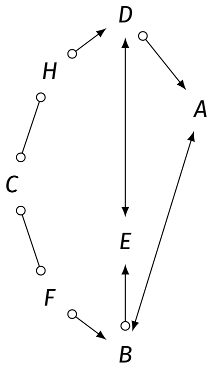


True graph



Inferred graph

Pros and cons of constraint-based methods

Pros:

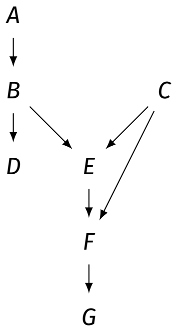
- Non-parametric (in practice, it depends on the selected independence test)
- Intuitive

Cons:

- Recover a partially oriented graph
- The faithfulness assumption is not always accepted

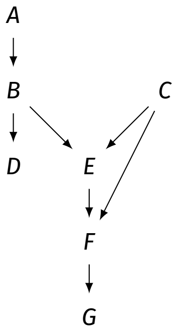
Exercise 1

What is the CPDAG of the following causal DAG?



Exercise 2

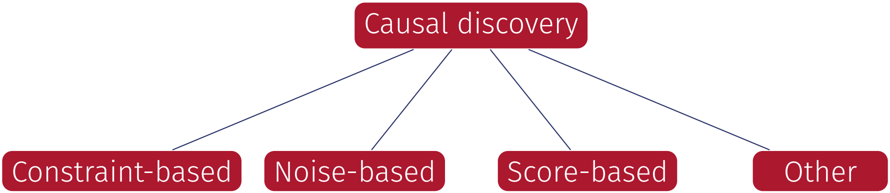
What will be the output of PC if $A = B$?



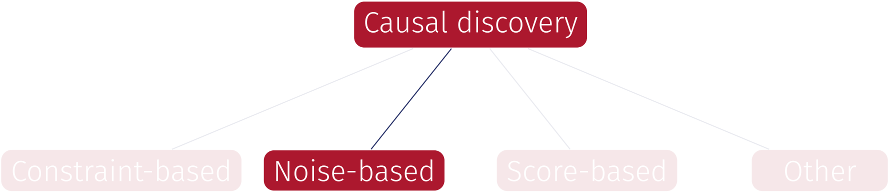
3

Noise-based causal discovery

Noise based


Causal discovery
Constraint-based
Noise-based
Score-based
Other

Noise based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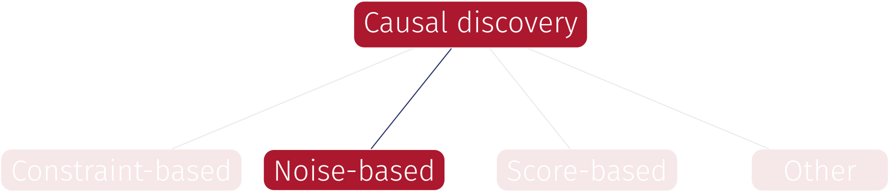
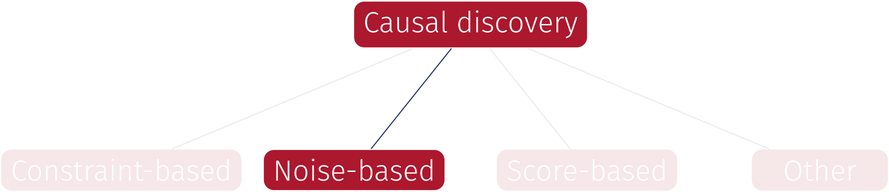


Noise-based: find footprints in the noise that imply causal asymmetry.

Noise based



Noise-based: find footprints in the noise that imply causal asymmetry.
Also known as semi-parametric-based or functional-based.

The intuition behind the noise (1/2)

$$\textit{Suppose} \begin{cases} X := \xi_x \\ Y := 2X + \xi_y \end{cases}$$

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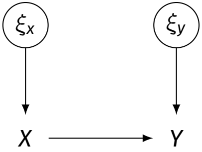
Assume that the noise follow a uniform distribution on $\{-1, 0, 1\}$

X	Y	$\xi_y = Y - 2X$	$\hat{\xi}_x = X - Y/2$
1	2	$0 \in \{-1, 0, 1\}$	$0 \in \{-1, 0, 1\}$
3	6	$0 \in \{-1, 0, 1\}$	$0 \in \{-1, 0, 1\}$
4	9	$1 \in \{-1, 0, 1\}$	$-0.5 \notin \{-1, 0, 1\}$

The intuition behind the noise (2/2)

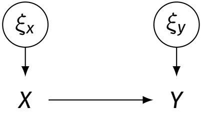


$$M_1 : \begin{cases} X := f_x(\xi_x) \\ Y := f_y(X, \xi_y) \end{cases}$$

- $X \perp\!\!\!\perp_{\mathcal{G}} \xi_y$
- $Y \not\perp\!\!\!\perp_{\mathcal{G}} \xi_x$

The experiment that changed everything

Suppose



$$X \sim N(0, 1)$$
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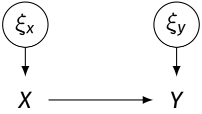

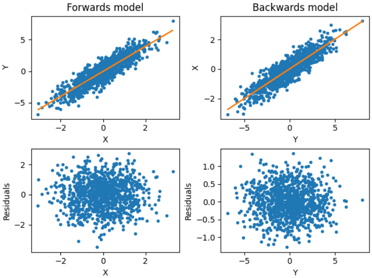

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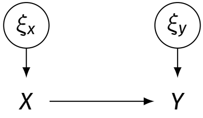

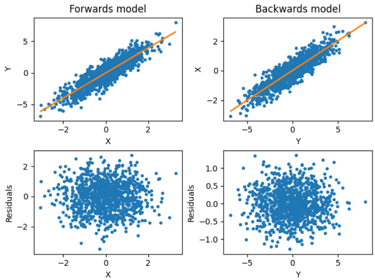

$$X \sim N(0, 1)$$
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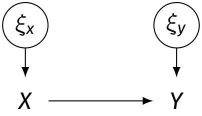

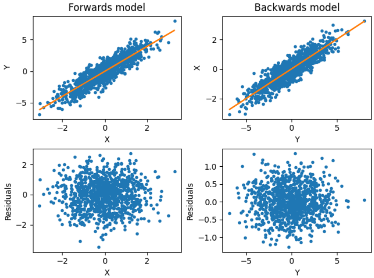

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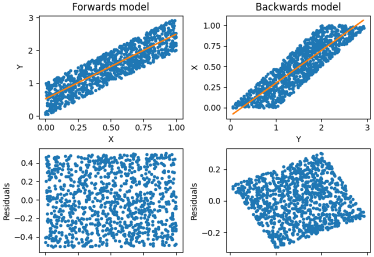

$$X \sim U(0,1)$$
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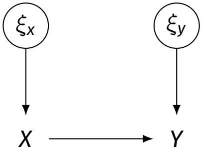
$$Y := 2X + \xi_y$$

The linear case (1/2)



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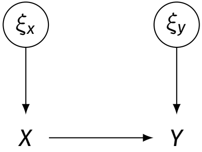
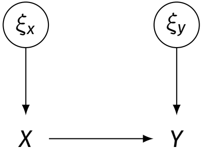


True model:

$$M_1 : \begin{cases} X := \xi_x \\ Y := aX + \xi_y \end{cases}$$

- $X \perp\!\!\!\perp_P \xi_y$

The linear case (1/2)



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Backwards model:

$$M_2 : \begin{cases} Y := \hat{\xi}_y \\ X := bY + \hat{\xi}_x \end{cases}$$

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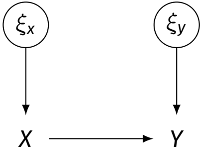

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- $Y \perp\!\!\!\perp_P \hat{\xi}_x$?

$$\begin{aligned} \hat{\xi}_x &= X - bY \\ &= X - b(aX + \xi_y) \\ &= (1 - ba)X - b\xi_y \end{aligned}$$

The linear case (2/2)

$$Y = aX + \xi_y$$
$$\hat{\xi}_x = (1 - ba)X - b\xi_y$$

When $Y \perp\!\!\!\perp_P \hat{\xi}_x$?

The linear case (2/2)

$$Y = aX + \xi_y$$
$$\hat{\xi}_x = (1 - ba)X - b\xi_y$$

When $Y \perp\!\!\!\perp_P \hat{\xi}_x$?

Theorem (Darmois-Skitovich)

Let $X_1, \cdots, X_n$ be independent, non degenerate random variables. If for two linear combinations:

$$l_1 = a_1 X_1 + \cdots + a_n X_n$$
$$l_2 = b_1 X_1 + \cdots + b_n X_n$$

are independent, then each X_i is normally distributed.

Theorem

Assume that $P(X, Y)$ admits the linear model

$$Y := aX + \xi_y, \qquad X \perp\!\!\!\perp_P \xi_y,$$

with continuous random variables X, ξ_y, and Y. Then there exists $b \in \mathbb{R}$ and a random variable $\hat{\xi}_x$ such that

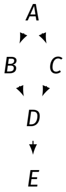
$$X := bY + \hat{\xi}_x, \qquad Y \perp\!\!\!\perp_P \hat{\xi}_x,$$

iff ξ_y and X are Gaussian.

Similar result for the multivariate case

A glimpse of LiNGAM in action

- Suppose the causal DAG on the right
- Input: Observational data
- Output: Causal DAG
- Assumptions: causal sufficiency, minimality, linearity, non-gaussianity



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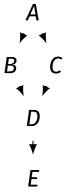
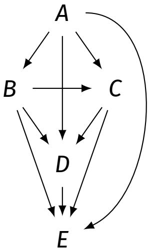
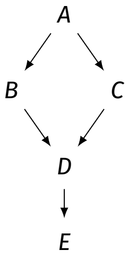


Causal order:



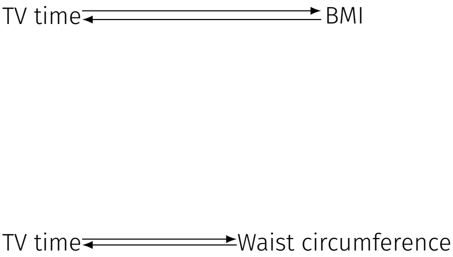
Pruning:



Real application of LiNGAM [4]

Young and middle-aged adults ($N = 2,060$); self-administered questionnaire for TV time

Specified graphs



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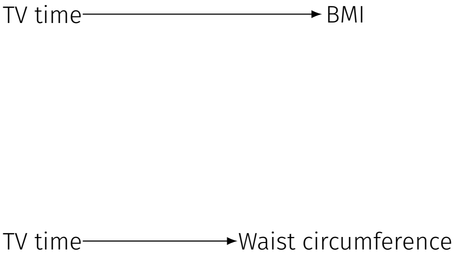


Extensions

- LiNGAM with hidden confounding
- Non-linear additive noise models:

$$Y = f(X) + \xi, \qquad \xi \perp\!\!\!\perp X.$$

- Post non-linear additive noise models:

$$Y = g(f(X) + \xi)\,, \qquad \xi \perp\!\!\!\perp X,$$

Pros and cons of noise-based methods

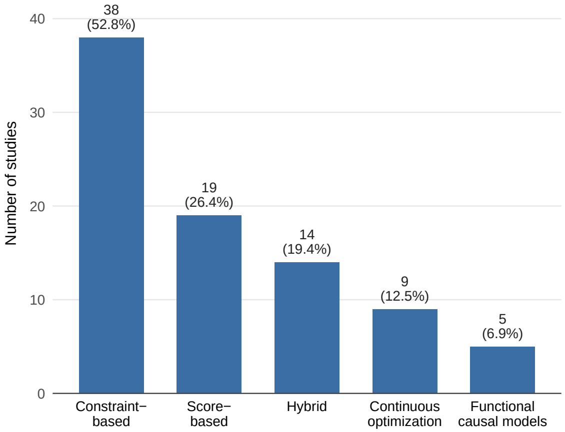
Pros:

- Recover the complete causal DAG

Cons:

- Semi-parametric

Causal Discovery in Observational Medical Research



1

[1]Liu, Zuting, et al. "Causal Discovery in Observational Medical Research: Scoping Review." JMIR Medical Informatics 14 (2026): e82499.

4

Practical considerations

Why are you using causal discovery?

What are the downstream tasks of causal discovery?

Why are you using causal discovery?

What are the downstream tasks of causal discovery?

- Understanding
- Identifying and estimating causal effects

Double-dipping

Causal DAG $\mathcal{G}$, dataset $\boldsymbol{D}$, $\beta_{\mathcal{G}}$ is the causal parameter of interest, then we have:

$$\mathbb{P}(\beta_{\mathcal{G}} \notin \boldsymbol{CI}_{\mathcal{G}}(\alpha, \boldsymbol{D})) \leq \alpha$$

What happens if I use the same data for the causal discovery and estimation?

Causal DAG $\hat{\mathcal{G}}$ estimated from dataset $\boldsymbol{D}$, dataset $\boldsymbol{D}$, $\beta_{\mathcal{G}}$ is the causal parameter of interest,

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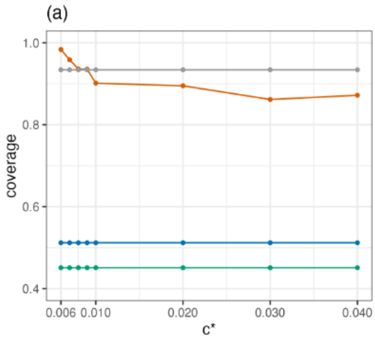
Double-dipping

Simulation experiment with tPC algorithm from [1]



Oracle
tPC + solution to double dipping
tPC, $\alpha = 0.01$
tPC, $\alpha = 0.05$

Possible solutions

- Split data into two datasets, one for causal discovery and one for estimation
- Resampling method [1]
- Noisy Causal Discovery [3]

Practical considerations

To be specified:

- The introduced implementation of PC is order dependent, however there exists order independent implementation introduced by [2]
- A conditional independence measure and test (constaint-based)
- An independece measure and tests (noise-based)
- A significance level in the statistical tests (constaint-based and noise-based)
- Optional: Existing expert knowledge (temporal ordering, forbiden orientations,...)

Usefulness of causal discovery when assumptions are violated

If assumptions are violated, then most causal discovery methods would give a Bayesian network
$\implies$ this Bayesian network could be used for the selection of the variables for the prediction task.

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5

Conclusion

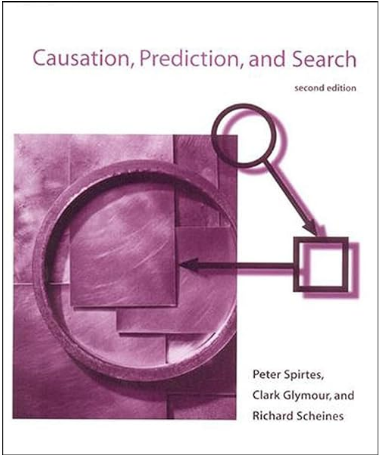
Conclusion

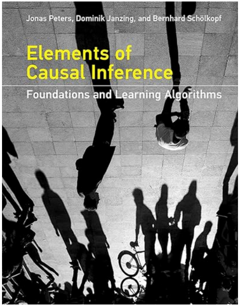
- In general, a causal DAG cannot be discovered from observational data alone.
- However, it can be discovered under certain untestable assumptions.
- If you can construct the causal DAG manually based on domain knowledge, then avoid using causal discovery methods (except for prediciton).
- Causal discovery is useful only when the causal DAG is unknown and needs to be inferred from data.

Causal discovery references

- "Review of causal discovery methods based on graphical models" by Clark Glymour, Kun Zhang, and Peter Spirtes,2019
- "Causal discovery algorithms: A practical guide" by Daniel Malinsky and David Danks, 2018
- "A practical guide to causal discovery with cohort data.",Andrews, Ryan M., Ronja Foraita, Vanessa Didelez, and Janine Witte,2021.

Causal discovery references





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